THE

Cause of the Supposed

PROPER MOTION OF THE FIXED STARS

AND AN

EXPLANATION OF THE APPARENT ACCELERATION OF THE MOON'S MEAN MOTION;

WITH OTHER GEOMETRICAL PROBLEMS IN ASTRONOMY HITHERTO UNSOLVED.

A SEQUEL TO THE GLACIAL EPOCH.

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PREFACE.

AT the commencement of the year 1873 we published a work entitled *The Cause, Date, and Duration of the last Glacial Epoch;* to that work the present volume is a sequel.

During the fourteen months that our former book has been before the public, and has been noticed by reviewers, we have been able to ascertain some of the opinions of those persons who have examined or read the book, and we have seen some of the criticisms that have been offered upon the problems contained in our work.

From what we have seen or heard, we have concluded that but few individuals appear to have completely understood the problem which was brought forward in our book; whilst some who have criticised the work seem to have entirely misunderstood not only the problem brought into notice, but the object of the book itself. In order to prevent any reader from continuing to misunderstand the problem, we will endeavour to explain it in as popular a form as the subject admits of.

When Copernicus brought into notice the theory—long before rejected by astronomers—of the rotation of the earth on its axis and its revolution round the sun, he also explained the cause of the precession of the equinoctial point by attributing it to a change in direction of the

axis of the earth. From an examination of the imperfect observations made at that date, it was supposed that the obliquity of the ecliptic never varied; consequently it followed that the angular distance between the pole of the heavens and the pole of the ecliptic, which angular distance is the measure of the obliquity, was always of the same value. It being a fact that the pole of the heavens changed its position as regards certain stars to the amount of about 20″ per year, and it being *supposed* that it never varied its distance from the pole of the ecliptic, it was concluded by former astronomers that the pole of the heavens must trace a circle round the pole of the ecliptic *as a centre.*

This geometrical explanation of the polar movement would be perfectly correct if the facts were such as formerly supposed. Granting that the pole of the heavens varied its distance 20″ annually from certain stars, but never varied its distance from the pole of the ecliptic, it would follow that the course of this pole must be a circle round that point as a centre from which it never varied its distance.

It having been supposed that the pole of the heavens traced a circle round the pole of the ecliptic as a centre, a theory was proposed to account for this movement, and it was agreed that the cause was the joint action of the sun and moon on the protuberance at the equator of the earth.

This theory, having been taught and elaborated for many years, came to be at length looked upon with a sort of religious veneration by its followers; so that when about a century ago observation seemed to indicate that there was a decrease in the value of the obliquity, these obser-

vations were rejected by many persons, and it was only after the facts became undeniable that certain theorists were forced to grant that there really was a decrease in the obliquity, which is the same thing as a decrease in the angular distance of the pole of the heavens from the pole of the ecliptic.

When it became known that there was a decrease in the distance of what was assumed to be the centre of a circle from its circumference, any competent geometrician ought to have at once seen that the old theory relative to the pole of the ecliptic being the centre of the circle traced by the earth's axis could not be correct. It appears, however, as if prejudice was then so deeply rooted, and geometry so little known, that in spite of the acknowledged fact that there was a decrease in the angular distance of the two poles, yet it still continued to be stated that the one pole traced a circle round the other as a centre, and never varied its distance from this centre.

A close examination of the writings of the past seems to give even more than a strong suspicion that geometry was formerly a science much neglected, and when theorists were fully occupied in finding theories to explain why the plane of the ecliptic varied the angle it made with the plane of the equinoctial, they omitted to notice that if these planes varied their distances it must follow, as a geometrical law, that the two poles must vary their distances, and that therefore the one pole could not be the centre of the circle traced by the other pole. Instead, however, of searching for the true character of the curve traced by the earth's axis, former theorists still adhered to the belief that the course of the pole of the heavens was

in a circle round the pole of the ecliptic *as a centre;* and so completely has this impossible theory taken root, that we find even now on every star-map a circle is printed on which is written, 'Circle traced by the pole of the heavens round the pole of the ecliptic at a constant distance of 23° 28′ during 25,840 years.'

That such a belief can be maintained when it is known that there is a decrease in the obliquity of the ecliptic is a proof that however profound may be the knowledge of those persons who adhere to this theory as regards the theories of physical astronomy, yet their acquaintance with the first rudiments of geometry and mathematics appears very slight; for divesting the question of its astronomical importance, it is merely stating that the centre of a circle can vary its distance from the circumference and yet still be the centre of the same circle.

Upon the supposition that the pole of the heavens traces a circle round the pole of the ecliptic as a centre, and completes this circle in 25,840 years, all modern calculations relative to the measurement of time, to the date of eclipses, and to the position of various celestial bodies, are based. It is therefore a problem of the gravest importance, demanding the strictest and most impartial inquiry.

Connected with the problem is another subject, which has also occupied the attention of mathematicians and astronomers for upwards of a century, viz. the so-called proper motion of the fixed stars.

In the present work, and in our last book, the problems dealt with were these:

First, it was demonstrated that the present accepted theory of the pole of the heavens tracing a circle round

the pole of the ecliptic as a centre is not correct, as it is not only not corroborated by facts, but is impossible from geometrical reasons.

Secondly, from an investigation of the curve really traced by the pole of the heavens it was demonstrated in our former work that the true curve was a circle having for its centre a point 6° from the pole of the ecliptic, and the discovery of this curve enabled us to arrive, by a purely geometrical calculation, at the value of the obliquity of the ecliptic for any year during the past two or three hundred years with an accuracy that amounted to a positive proof.

Thirdly, when we carried on the same curve for 13,000 years, that we found had been traced for 300 years, we discovered that it must have caused on earth a climate of an arctic character down to 54° of latitude in both hemispheres.

When we search the recorded facts of geology, we find that there was just previous to the present conditions exactly such a climate on earth as would result from the polar movement we had discovered, and what was equally as convincing was, that geology demonstrated that this climate did not extend lower than that parallel of latitude indicated by the polar movement to have been the boundary of the Arctic Circle.

It must be borne in mind that the movement of the pole of the heavens which we have demonstrated is not a mere theory invented to explain a particular climate on earth, but the movement is proved by the recorded observations of at least 300 years, if not of 1800 years, and is *corroborated* by the evidence of geology in a manner perhaps more complete as regards its details than could

be supposed possible, considering the facts occurred so many thousand years ago.

These facts, however, being indicated by the evidence of vast boulders, by accumulated masses of drift materials, by the scraping of vast continents, by icebergs, by the cuttings of valleys by means of giant streams, and by every other form evincing a period of ice action, are of so formidable a character that they cannot be avoided or denied. That these facts have not hitherto received a full and satisfactory explanation seems mainly in consequence of those theoretical authorities on astronomical matters, who claimed to deal with the most exact of sciences in the most accurate manner, contentedly accepting the impossible theory that the centre of a circle could vary its distance from its circumference and yet could still remain the centre of the same circle.

The object, then, of our work, *The Cause, Date, and Duration of the last Glacial Epoch*, was—

First, to point out and demonstrate that the present accepted movement of the pole of the heavens was not only incorrect, but could not possibly occur.

Secondly, to demonstrate what this movement really was, and to show how results might be calculated which hitherto it had been deemed impossible to arrive at by calculation.

Thirdly, to demonstrate that the true movement of the pole of the heavens was such as to fully explain the peculiar climate on earth known to exist during what has been termed the Glacial Epoch, and to give the date and duration of this remarkable period.

Fourthly, to call attention to some of the astronomical problems which would be dependent on this movement of

the pole of the heavens, and to hint at other geological results which might be due to the same cause.

These being the problems brought forward in our late work, we may now refer to some of the remarks which have been made by reviewers, and which we believe must have arisen from these writers not having understood the problems in the said work, or from their not having been able to devote sufficient time to the study of the work before they favoured their readers with a critical notice of it.

Among the first notices of our book was one in which the critic informs his readers that our object in writing the book was to prove the earth was not round, but was a flat surface, and that we had a quarrel with the 'elliptic.'

What the reviewer meant by stating so incorrect an assertion we are at a loss to guess; and what he meant by the 'elliptic' we can only guess. He may have meant an elliptic orbit, or he may have meant the ecliptic; but to gravely write in this manner of a problem of importance is scarcely likely to add to the reputation of a reviewer.

Several notices of the book have been ably and carefully written, but there has been generally a tendency to overlook the main problem at issue, and to refer to some minor item in geology, and to point out that the movement of the pole of the heavens which we demonstrated, would not explain this one item, and was not therefore satisfactory to the critic.

Thus one reviewer points out that the alternate beds of coal, sandstone, and shale are not satisfactorily explained by this theory. In reply, we can state that we only hinted at the possibility of the polar movement

giving a clue to the cause of this alternation. Again, it was objected that the alternate elevation and depression of land was not explained by our theory. We grant it is not explained, neither is the cause of an earthquake explained, nor of cholera, nor of the aurora by this movement of the pole; and we have not attempted to explain such events. That which we do claim to demonstrate is, that the pole of the heavens does not trace a circle round the pole of the ecliptic as a centre, but that it does trace a circle round a point 6° from the pole of the ecliptic, and this course does fully explain the climate of the last Glacial Epoch.

Some reviewers state that it would be impossible that the limited time which we have given was sufficient for the accumulation of the drift materials of the Glacial Epoch.

Considering that upwards of 16,000 years of arctic winters and tropical summers would have elapsed, we think the cause ample for the effects. Other objectors have stated that the theory was untenable, as the earth has existed only 6000 years.

In more than one notice the remark has been made, that however anxious geologists might be to accept such an explanation of the important facts of the glacial period, yet they must wait for the verdict of astronomers.

Whilst fully agreeing with the importance of this remark, there are reasons why it does not carry with it all the weight that it at first sight appears to do.

If by the term 'astronomer' was meant a thorough geometrician and mathematician, who had studied all the movements of the celestial bodies, and had not only learnt the rules and theories that have been hitherto supposed correct, but was so unprejudiced as to be willing and able

to devote his time and thought to the fair examination of a new problem, and was able to fairly weigh the merits of this problem in comparison with what had hitherto been supposed satisfactory, then we admit that the verdict of such an astronomer, *if in accordance with facts,* would be worthy of all consideration.

When, however, we find that there are certain gentlemen termed 'astronomers' and 'astronomical authorities' whose claims to these terms seem to consist entirely from the fact that they have been able to purchase large telescopes, have had sufficient time at their disposal to look through these and to publish what they have seen, and have thus accumulated volumes of recorded right ascensions and declinations, we do not consider that the off-hand opinion of such gentlemen on a geometrical problem which is clear and simple should be considered finite, when it is opposed to the facts that are brought forward.

Again, when we find that an astronomical authority who is a geometrician and mathematician considers that he adduces a sound argument against our theory when he states that 'the rule for finding the obliquity of the ecliptic is to subtract $0''\cdot45$ per annum, and that this rule works very well, *and therefore* our polar movement is not needed,' we perceive that the idea present in this authority's mind is merely how to patch up records, so as to be able to predict them very closely for a few years in advance.

This is the system which we find has been practised in connection with 'time' and the value of the obliquity of the ecliptic, quantities depending entirely on the correct course of the pole of the heavens round the pole of the ecliptic; for it was found necessary to fudge-in 3^m

$3^s.68$ in 1833 to make facts and theories agree. Also, between 1861 and 1865 there is a difference of $2''.23$ in the obliquity, which gives $0''.557$ per annum, as the decrease between these dates, in order to make facts and theories agree, instead of $0''.45$.

Considering the importance of the results depending on this problem, it may fairly admit of question whether it is one that ought to be decided and disposed of by any small body of individuals who have previously agreed that their theories are perfect, especially when we find that these gentlemen have, during thirty years, accepted and agreed to the belief that the centre of a circle can vary its distance from its circumference and yet remain the centre, and have supposed that there was no imperfection in a science which left the vast climatic changes of geology unexplained. Also, we doubt the capacity of such gentlemen as competent critics on a geometrical question when we find in their writings* that they adopt for discovering the supposed proper motion of the fixed stars a formula that is in its first principles unsound, and does not give the proper motion of the stars, but merely results of a discordant nature due to the erroneous principle they have adopted.

If any person considers this question fairly, he will understand that we are making a very large demand on an acknowledged authority, who, for say thirty years, has been considered infallible, when we expect him to examine a novel problem and to acknowledge that this problem is correct, and that he himself has for thirty years been in error, has taught that which was wrong, and has based his calculations on that which was incorrect. Those

* See pages 142 to 150.

reviewers who state that they must be bound by the verdict of astronomical authorities assume that such an acknowledgment will be made supposing our problem is correct, whereas the whole history of astronomy yields no example of such a thing. The authorities on astronomy and geography in the days of Herodotus opposed and ridiculed the idea of the earth being a sphere. Astronomers in the days of Pythagoras rejected the theory of the movement of the earth, as they did again in the days of Copernicus and Galileo. It was the authorities in Galileo's time who pronounced that Jupiter could not possess satellites, as they must be useless; and these objections were raised, most probably, not because the objectors meant to wilfully oppose that which was true, but because the very long time they had been accustomed to accept an old theory as correct, rendered them incapable of fairly examining a new problem. If a reviewer in former times had written that it must be left to Sizzi and the other professors of Padua to decide whether or not Jupiter possessed satellites, and whether the earth rotated on its axis, we believe we could predict what the verdict would have been; and it remains to be proved whether the human intellect and human prejudices in 1874 are very different from what they were in 1633.

From a consideration of these facts, we believe that those reviewers who have intimated that they must be bound by the decision of that large class designated by them 'astronomers' will, on reconsideration, find that their opinion requires some modification.

In some notices of our former work we find excellent examples of the style of reasoning and of argument urged by ancient authorities against the, to them, ridiculous

theory of the earth being round. In a paper which professes to devote itself to scientific subjects, we find the reviewer commences his notice with some assumptions of his own relative to our ignorance of dynamics, and then informs his readers that we are attempting to solve a problem by the aid of mere geometry, as if such a statement were a quite sufficient condemnation of the problem.

This writer has evidently either not read our work, or, having read it, is so unacquainted with this science of geometry which he despises, as not to perceive that he grants the truth of our problem when he announces what he thinks a contradiction, but which in reality is an acknowledgment of our facts. Having asserted that it is not believed by astronomers that there is any decrease in the obliquity of the ecliptic—an assertion as startling to an astronomer as to state that we do not believe the earth rotates on its axis—this writer announces the following, evidently with the idea that it is a sufficient answer to our work:

'The popular belief is that the pole of the earth describes a circle of radius 23° 28′ round the pole of the ecliptic as a centre, and that the whole circle would be described in something over 25,000 years.'

This popular belief, as the writer terms it, is the theory which any person versed in the mere elements of geometry at once perceives is impossible, when it is known that the obliquity of the ecliptic is now decreasing 0″·45 per annum, and that a decrease has been going on for at least 2000 years. As, however, the writer of the above notice denies that such a decrease occurs, we need not farther refer to his somewhat curious description of what he supposes our problem is.

When it was announced formerly that the earth was not a flat surface, but was round like a ball, the authorities at that remote date produced a ball, and demonstrated that if the earth were round all the water would run off it the same as off the ball, and that people underneath must be walking like flies on the ceiling, which was, of course, absurd, and therefore the earth, they said, was not round. It is the fashion in modern times to ridicule these ancient authorities, and to consider them weak-minded, instead of really weighing the difficulties they encountered in getting out of their old-fashioned grooves and fairly grappling with a new idea. It does, however, seem singular to find in a notice of our last work an argument brought forward as a reason for our problem being incorrect, which argument is exactly similar to that relative to the ball proving the earth cannot be round.

An argument brought forward against us is, that a boy's top, when spinning round an inclined axis, will also revolve round a vertical axis with a slow motion; consequently, says this writer, unless a top is produced which will do what the earth is supposed to do, he will feel confident in rejecting the theory of the earth's motion.

This fossil argument seems to be brought forward as not only powerful but original; it is neither: just as a ball does not exhibit the same conditions as the earth with its rotating motion, its revolution round the sun, and its attracting power, so a boy's top, or even a gyroscope, does not present the same conditions as the earth, and the comparison between the two is almost as out of place as was that comparison supposed to controvert the theory of the earth being round.

It is gratifying to find that the reviewer who brings

forward the above little demonstration as a proof that our problem cannot be true considers our work 'most unsatisfactory,' and believes that it is a fatal error to bring 'mere geometry' to bear on an astronomical problem. We can quite understand that this writer, who is but a representative of a large school of so-called philosophers, would prefer something less exact than geometry to be brought against him, and some theory (more resting on strongly expressed opinion than on facts), such as the ball or the top theory, on which to argue that the belief he had so long indulged in was perfectly sound.

Any person really acquainted with dynamics will at once perceive the inefficiency of these sketchy arguments, and the erroneous assumptions that have been taken as the foundation of the objection. A boy's top is wound up with a string, is then hurled on the ground, and is thus spun, and performs a certain movement. The earth rotates on its axis also, consequently, says the tyro in dynamics, all the movements that the top makes must of course be the same as the earth makes, the pole of the earth must trace a circle round a centre from which centre it never varies its distance, and this theory is so profound, urges its devotee, that mere geometry must not be brought forward to expose its absurdity.

In the first place, the earth is suspended in space, and does not rotate on a peg, which peg rests on a plane; unless, therefore, a top can be suspended in space whilst rotating, the two conditions are in no case alike. If a rotating top be thrown up in the air, the axis does not gyrate in the same manner as when the peg is resting on a rigid surface. A mechanician will of course see the reason for this.

Secondly, are we sure that the rotation of the earth on its axis is due to its having been wound up and spun at some former period? May not the earth's rotation be due to a more subtle cause? Unless a theorist can demonstrate that the earth's rotation has been produced by its being spun by mechanical means on its axis, and unless he can show some greater similarity between a top spun on its peg and the earth rotating and revolving in space, we must point out that his reasoning belongs to a past century, and is of the same class as is the reasoning of those who declined to accept the spherical form of the earth because water would not stop on a ball.

Having, however, pointed out that the comparison between a spun top and a rotating planet is not an exact one, and that the movements of the one need not be followed exactly by those of the other, we can now state that a gyroscope top, when spun and placed on a rest, with its axis inclined at an angle to the vertical, does move in exactly the same manner as we have stated the earth moves. The axis of the rotating top does describe a circle. We state the axis of the earth describes a circle. The axis of the rotating top describes this circle round a point as a centre, from which centre it never varies its distance. We demonstrate that the earth's axis describes its circle round a point in the heavens, from which point, termed by us the centre of polar motion, it never varies its distance. Thus the movement described by the top is similar to that which we have stated the earth makes.

Let us now point out the unsound reasoning of the critic who thinks our demonstration unsatisfactory. He concludes that a top must gyrate round a centre which is 23° 28′ only from the circle described. We state that re-

corded facts prove that the earth's axis gyrates round a centre 29° 25′ 47″ from the circle described. Hence the two movements are exactly similar in all respects, except as regards the angle which the axis of rotation makes with the line, or axis, of the cone traced by the conical movement of the rotating body. The position, then, taken by this writer is, that because the axis of a top traces a conical movement, therefore the earth's axis must trace a circle round a point *exactly* 23° 28′ *from the circumference*, and that any other radius for this circle than 23° 28′ is impossible.

We believe there is no difficulty in demonstrating that the axis of a rotating top will trace a conical figure at any angular distance from the vertical, and its movement in this form is not limited to the special angle of 23° 28′; and we are therefore quite prepared to demonstrate that a top does move as we have shown the earth moves, though we place little value on this experiment, because all the conditions are not the same as regards a top and the earth.

When we find that the course traced by the pole of the heavens during 300 years is a curve such as we have defined it, and that this curve enables us to calculate with minute accuracy for all time the value of the obliquity, a calculation which has never before been possible, and that geometry enables us to accomplish this, we may well decline to accept as valuable the mere opinions of those who sneer at geometrical science, who indicate that they have overlooked the principal facts announced in our works, and who assert that no astronomer believes that the obliquity of the ecliptic changes.

It is in consequence of this sneering at and neglect of geometry that has caused speculative theorists to over-

look the fact that for a hundred years they have accepted as true two theories which contradict each other. To a person unacquainted with geometry there seems nothing unsound in stating that the centre of a circle can vary its distance from the circumference and yet still always remain the centre, and this is the statement now put forward as correct by certain theorists. That a curve is demonstrated by geometry to be of a particular character is no more proof to a person unacquainted with geometry than would be the evidence to a child that the three angles of a plain triangle must amount to 180°. To a person thus unacquainted with the most exact of sciences, it would appear more convincing to compare a ball or a top to the earth, to assume that each and every one of the conditions affecting the ball were also exactly similar to those affecting the earth, and then to consider it proved that whatever happens to the ball or top must happen to the earth. Q. E. D.

It is most singular to note how permanent a hold this unsound system of comparison has on certain minds. Two hundred years ago it was employed against the fact that Jupiter had satellites. It was then argued that as the universe was exactly represented by man, and as man had two eyes, two ears, two nose-holes, and one mouth, making seven in all, so there are the sun and moon, Venus and Mercury, Mars, Jupiter, and Saturn, making seven in all. Therefore, it was urged, there could not be any satellites to Jupiter, otherwise the order of nature would be upset.

There is another argument used more than once by those critics, who were evidently more acquainted with the facts of geology than with the laws of geometry.

This argument is, that there is some theory already which in their opinion seems to explain the climatic changes on earth; and they have urged, therefore, that what they term this 'new theory is not required.'

Now, it is not a question whether a new theory is or is not required, or whether any old theory does or does not explain the climatic changes on earth (we believe that no previous theory has ever explained the recorded facts); but it is a question whether it is sound geometry, and possible, for the centre of a circle to vary its distance from the circumference and yet remain the centre. If such a change be not possible, then the present accepted doctrine relative to the movement of the earth's axis is incorrect and impossible.

Having found that the present accepted movement is impossible, it follows that some other movement must occur, and it is then a question whether our evidence proves the movement to occur which we have brought into notice.

To urge that the so-termed 'theory' is not required, because some theory now believed in is supposed to explain a few phenomena, would be similar to stating that the theory of the earth being a sphere is not required, because the theory of its being a flat surface explains some few phenomena.

It is not a question of whether some old theory explains certain effects, but what is the real movement of the earth's axis as regards the sphere of the heavens. In the present volume we give additional evidence of what appears, from recorded observation, to be the course of the pole of the heavens relative to the pole of the ecliptic. We then bring forward some original problems

connected with the changes of stars' positions as produced by the polar movement, and demonstrate how impossible it is to predict what a star's change in right ascension and declination really should be when we do not know the real movement in the pole of the heavens.

We next demonstrate that the system hitherto adopted and supposed to give the proper motion of the fixed stars does not give this proper motion, because the system adopted is geometrically unsound, and we show that the point in the heavens hitherto supposed to be that point towards which the whole solar system is moving, and which has been termed *the apex of solar motion*, is in reality that point in the real circle traced by the pole of the heavens which is farthest removed from the pole of the ecliptic.

We believe it will be granted that this one problem is of sufficient importance to justify the publication of a book to contain it, even if no other original problems were submitted therein.

The conditions relative to the changes in various meridians due to a conical motion of the earth's axis are also described, and it is pointed out that for four almost identical movements of the earth's axis there may be four entirely different movements of the earth itself. This fact is, we believe, new to astronomers, and the results which follow have never hitherto been examined.

The problem of the supposed acceleration of the moon's mean motion is, we demonstrate, dependent on this motion of the earth's axis. This problem has not only hitherto not been solved, but has never been approximately solved, from the fact that theorists have hitherto searched for some physical cause to explain why

the moon did move more quickly, instead of even suspecting that our present standard of time was not a constant quantity.

We believe that when astronomers and geometricians do turn their attention to these facts, they will soon discover their importance, and will then not need to be told that they supply the key to much more than is here even mentioned.

In our work, the *Glacial Epoch*, we promised that additional evidence would be given to prove that the course of the pole of the heavens was not such as had hitherto been supposed, and we stated we would show a cause for the so-called proper motion of the fixed stars. In the present work these problems are both dealt with, though much more remains to be said than could be inserted in the brief time at our disposal.

CONTENTS.

ON THE

MOTION OF THE FIXED STARS.

CHAPTER I.

LATITUDE AND LONGITUDE OF STARS, AND THEIR RIGHT ASCENSION AND DECLINATION.

One of the principal objects of practical astronomy is to obtain such a knowledge of the mechanism of the heavens as to be able to predict the position of any celestial body for any time. Long before it was even supposed that the earth rotated on its axis, the Greek, Egyptian, and Hindu astronomers had, by observation, become acquainted with certain changes in the relative positions of various celestial objects, and had so tabulated these changes, that they could predict with tolerable accuracy the relative positions of the moon and the stars, and of the planets, for long periods in advance.

There is nothing more simple, and yet few things which produce more wonder on uneducated minds, than the accuracy with which eclipses are foretold. If we are to believe the history of the adventures of

Columbus, we must credit that his knowledge relative to a coming eclipse was one cause for his obtaining an influence over the savages of America; whilst in many other recorded instances men who have possessed a little knowledge on this branch of astronomy have traded on it, in order to induce the ignorant to believe their wisdom was far greater than it really was, and to obtain for themselves such faith that they were believed in, when, in reality, they were merely themselves guessing.

Eclipses both of the sun and moon being matters visible to the inhabitants of a large portion of the earth, of so remarkable a character as to attract general attention and to be recorded in history, it would require merely average intelligence and an examination of records to reveal the fact that eclipses recurred in a regular order. As an eclipse is due to the fact of the sun, the moon, and the earth being, as we may term it, in the same straight line, it follows, that if we find in what manner the moon appears to travel relative to the sun, and if we discover that the moon, during a certain number of years, returns exactly to the same position, and then repeats its movements in the same manner over and over again, then eclipses will recur after certain intervals; and hence we can predict them.

This was really the case with regard to the ancients; they found that the moon performed a 'cycle,' as we may term it, and after about 18·6 years came back to the same position relative to the fixed stars.

Consequently, although the men of that day knew not that the earth rotated on its axis, and that the rising and setting of the celestial bodies was due to this rotation—and although they did not know that the earth revolved round the sun, and thus produced most of the other changes in the celestial bodies—yet they could predict an eclipse with tolerable accuracy, and could also foretell when the moon would pass before certain stars.

We must call particular attention to these facts, because they demonstrate that there are various results which occur in connection with astronomical science and which can be foretold, and about which nearly every detail is known, and yet the actual cause of these movements may be as unknown to the recorders of them as the ancients were ignorant of the cause of the apparent rising and setting of the celestial bodies.

Astronomical science may fairly be divided into many branches. Two of these may be classed as the *observational* and the *geometrical.* Under the head of the observational may be gathered all those items connected with mere observation, such as noting the relative positions of various celestial bodies; whilst under the head of geometrical we may class the reasoning upon, and investigation of, these observations, with a view to the discovery of those dynamical powers by which the worlds around us are moved.

Mere observation can never arrive at any result until the whole cycle, and perhaps many cycles,

have been observed. For example, if the sun's mid-day altitude were observed on the 1st of January of any year, and again on the 1st of February and 1st of March, observation alone could tell us nothing more than that there was a certain increase in this meridian altitude. Geometry, however, could analyse this rate of increase, and would probably be able to predict what would be the sun's meridian altitude for every day in the year.

There are several items connected with astronomical movements which recur with such rapidity that observation alone enables us to know nearly every detail connected with them. The rotation of the earth, for example, which occurs about 366¼ times in one year, is a movement well known. The revolution of the earth round the sun during a period we term a year is another such movement. A completion of a lunar cycle during about 18·6 years is again another such event of which we know the details, because all these have been observed. When the whole course of any of these events has been completed, we know what this course is from observation; and even though we attribute this course to an incorrect cause, we do not obtain incorrect results, because our results are already known from observation. When, however, we have to deal with certain movements which occur so slowly that hundreds of centuries are required to enable one cycle to be completed, we do not from observation alone know the details of this cycle; and if we desire to know what these

details are, we must resort to geometry and reason, and endeavour to trace the unknown portion of the cycle from the data relative to the known. If we can accomplish such an end, we in a brief lifetime live, as it were, thousands of years, whilst the mere observer lives only two or three score years.

Among those problems which have occupied the attention of astronomers from the earliest dates none have required more care and more attention than that of predicting the exact position of every star in the heavens for long periods in advance. It has been often supposed that all the difficulties of this problem had been surmounted, and that at last the real cause of every inconsistency between facts and theories had been discovered; but the real movements of the stars relative to certain fixed or movable points has been such, that the sanguine expectation of theorists has hitherto been doomed to disappointment. The reason is, that a movement is occurring which causes the positions of every star, as referred to certain datum-points, to change, and this movement is so slow that many thousands of years are required to complete one cycle. Thus man, from observation alone, does not know of what this complete cycle consists, and he has been driven to speculate as to what it is. The conclusions to which he has come have been based upon certain observed facts, and his conclusions are consequently open to criticism, because we have the facts, and we know what the conclusions are, and we can compare the two.

In order to investigate fully the problem relative to the positions and supposed changes in the stars, we must give a brief history of the various methods adopted from the earliest ages, in order to map-out the stars in their relative places in the heavens.

There are two great circles in the heavens, to one or other of which the stars have at all times been referred; and there is one point in the heavens which has at all times been used as a point of reference.

One of these circles is termed the plane of the ecliptic, and it is that circle in the heavens along which the earth would appear to travel if seen from the sun. By merely interpolating, we obtain this circle by noting it as the course of the sun among the fixed stars as seen from the earth. As every circle has a pole or centre, the plane of the ecliptic has a pole, distant, of course, 90° from all parts of the ecliptic.

The other circle is that traced by producing the plane of the earth's equator to the fixed stars, and thus marking out an imaginary circle in the heavens. This circle or plane of the equinoctial is distant 90° from the pole of the equinoctial, or pole of the heavens, as it is termed; this pole of the heavens being that point in the heavens towards which the earth's axis is directed.

These two circles or planes form with each other an angle, which is equal to what is termed the obliquity of the ecliptic; and the two poles are separated

from each other by an angle equal to the inclination of the one plane to the other.

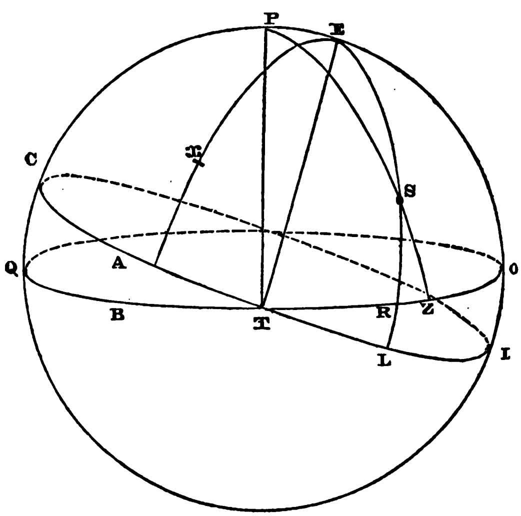
It is to one or other of these planes that the positions of stars have been referred, from the very earliest down to the most modern dates. The system of placing these stars has, however, undergone considerable change in modern times, producing results of a singular and valuable kind. In the most remote ages the stars were assigned positions relative to the plane of the ecliptic, these positions being described as their latitudes; and their longitudes were measured by the angle formed at the pole of the ecliptic by two meridians, one of which passed from the pole of the ecliptic through the star, the other from the pole of the ecliptic through that point of the ecliptic where it intersected the equinoctial.

Upon referring to the diagram page 8, we can show how a star's position is indicated by its latitude and longitude.

The sphere shown in this diagram represents the sphere of the heavens. The sun's course on that sphere is represented by the circle C A T L I, which is the ecliptic, the pole of the ecliptic being at the point E. The circle Q B T R O is the equinoctial, the pole of which, P, is called the pole of the heavens. The angle formed by the two planes Q B T R O and C A T L I, and measured by the spherical angle Q T C, is the measure of what is called the obliquity of the ecliptic. This angle is also measured by the value of the arcs Q C or I O, and by the angular distance, P E, of the two

poles. The obliquity of the ecliptic, as it is termed, gives the angular distance that the sun appears to travel annually north and south of the equator, and thus indicates the extent of the tropics.

It is a geometrical law that just as far in angular distance as the sun appears to travel south of the equator, so far will the arctic circle extend from the north pole; and just as far as the sun extends north of the equator, just so far will the antarctic circle extend from the south pole. Thus the angular distance Q C is the measure of the tropics and of the arctic circles. The point T, where the two circles intersect, is termed the equinoctial point; and it is that point in the heavens where the sun's centre would be located, if it could be seen, on the sphere of the heavens about the 21st of March, when its centre was vertical at the equator.



If we join E, the pole of the ecliptic, with T, then E T is the zero meridian from which a star's longitude is counted; whilst its latitude is counted from the circle C A T L I on circles towards E. Thus, a star situated at S would be assigned a latitude S L, and a longitude measured by the angle T E L; and as the arc T L is the measure of the angle T E L, the longitude of the star S may be given as T L.

Longitude was usually counted by signs of the zodiac, and then so many degrees of that sign in which the star was located, or it was counted round the whole 360° of the circle. Thus, a star at X would have assigned to it a latitude of X A, and a longitude T E A, counting round from left to right; thus this star would have a longitude of 360°—the angular distance T A.

In the four earliest catalogues of which we have any record, the positions of the stars were assigned by their latitudes and longitudes. Considering that the ancient astronomers had no clocks, and only very inferior instruments, it may be of interest to some of our readers if we describe the method adopted by the ancients to determine these latitudes and longitudes of stars.

By the aid of a vertical circle placed in the meridian, the greatest and least altitude of the sun could be noted during the year. The point between those indicating the greatest and least altitude would indicate the sun's altitude when it was on the equator. Then, knowing that the sun moved over a certain arc

each day, the sun's angular distance from the point at which the equator and ecliptic intersected would also be known; and this angular distance was the sun's longitude.

As the stars could not be seen whilst the sun was visible, the ancient astronomers were compelled to have recourse to another aid in order to determine longitudes, and they employed the moon as a sort of go-between. Having found the difference in longitude between the sun and the moon when both were visible, they could find the difference in longitude between the moon and a star, because they knew from observation the amount of movement of the moon in a given time, and could allow for this. Hence, to determine a star's longitude with some approximation to accuracy, even without clocks or telescopes, was quite within the power of the ancient astronomers.

It was in consequence of such observations as these having been made by ancient astronomers that the precession of the equinoxes, as it is termed, was discovered. When Hipparchus repeated this operation, and compared the results which he obtained with those which had been arrived at by his predecessors, he found that every star had, in his time, more longitude than it had formerly; and as he placed faith in the observations of the former observers, he concluded that there was some movement taking place in the sphere of the heavens which produced this result.

This discovery was one which we may term of a purely observational nature; it required but little

geometrical knowledge, but was arrived at much in the same manner as we could tell that a tree had grown. So many writers have spoken of this original discovery as so very grand, and to exhibit so great an intellect, that, without attempting to underrate its practical importance, we cannot refrain from pointing out that it is scarcely equal in difficulty to, and requires no more profound knowledge than does the discovery of a comet, or ascertaining that certain coast-lines have altered their form as shown by ancient maps, or that various volcanoes have altered the form of their craters.

From its having been supposed that every star, whilst it maintained apparently the same relative distance from all other stars near it, increased uniformly its longitude whilst its latitude remained constant, it was assumed by the ancients that the sphere of the heavens (to which they believed the stars were attached) slowly rotated round the pole of the ecliptic. They even went so far as to make a close approximation to this rate of apparent rotation; but for many reasons their accuracy was not very great.

Four catalogues of stars, in which their latitudes and longitudes are given, are in existence, viz. a catalogue of Ptolemy's, dated 140 A.D.; one of the Arabian Prince Ulugh Beigh, dated 1437 A.D.; Tycho Brahè's catalogue, 1628 A.D.; and a catalogue by Hevelius of a later date. These four catalogues are published in vol. xiii. *Memoirs of the Royal Astronomical Society.*

These catalogues, although valuable in some respects, cannot be accepted as very accurate. Ptolemy did not attempt to indicate a star's position more nearly than $\frac{1}{6}$th of a degree, which seems to prove that his instrument did not read to a smaller angle than $\frac{1}{6}$th of a degree. He was also unacquainted with the effects of refraction, and thus his latitudes would be slightly in error, especially for those stars near the horizon. Then, again, as the longitudes were determined by aid of the moon, the moon, when near the horizon, would have a large parallax, and would thus appear higher in the heavens than she ought to be; therefore her angular distance from a star would be assigned, in the majority of cases, greater than it ought to be. All these catalogues deserve to be considered as approximations only, and not as correct records, of stars' positions.

When telescopes were invented, another method was adopted to fix the positions of various stars, and their right ascensions and declinations, or north polar distances, were then usually assigned. The declination of a star is its least angular distance from the equator, whilst its right ascension is the angle formed at the pole of the heavens by two meridians, one of which passes through the first point of Aries, the other through the star. Thus, referring to diagram on page 8, a star, S, would be assigned a declination of S Z or a polar distance P S; and as P Z is 90°, the polar distance is the complement of the declination. The right ascension of this star would be measured by the

angle at the pole, termed the hour angle, formed by the meridians P T and P Z, and this angle can be measured by the arc T Z of the equinoctial.

If we know the latitude and longitude of a star, and the obliquity of the ecliptic, we can calculate its right ascension and declination; or, if we know its right ascension and declination, and the obliquity of the ecliptic, we can calculate its latitude and longitude.

For example, suppose we know the latitude and longitude of the star S, and the obliquity of the ecliptic, we then know E S, the colatitude, equal to 90°—the latitude; we know E P, the obliquity, and we know the angle P E S=90°+T E S=90°+the longitude of S.

Then with the two sides E P and E S, and the included angle P E S, we can find the third side P S, which is the polar distance of S, and equal to the complement of the declination. Having the two sides, and the included angle as above, we can find the angle E P S, which is equal to 90°—the right ascension of S. Thus from a catalogue of longitudes and latitudes we can calculate a star catalogue of right ascensions and declinations.

The most convenient formula for this purpose is the following:

$$\text{Cot. } \alpha = \text{sin. L cot. l}$$

$$\text{tan. AR} = \frac{\text{cos. } (\alpha + \omega)}{\text{cos. } \alpha} \text{ tan. L}$$

$$\text{tan. D} = \text{tan. } (\alpha + \omega) \text{ sin. AR}$$

Where L=Longitude ω=obliquity of Ecliptic
l=Latitude
AR=Right Ascension
D=Declination.

The modern astronomical observer has the advantage, which the ancients did not possess, of telescopes and clocks, and he can accomplish with ease that which the olden observers could only arrive at with difficulty. Hence in modern times forming a star catalogue is comparatively a simple operation. The following is a brief description of the method now adopted for forming a star catalogue:

An instrument, termed the 'Transit Instrument,' is placed so that it revolves exactly in the meridian, therefore also it points, when vertical, exactly towards the zenith. By aid of this instrument the latitude can be determined before the declination of any star is known, by taking the mean of the upper and lower culmination of a circumpolar star; or the latitude may be obtained by ascertaining the greatest and least altitude of the sun during the year. Making the proper allowance for the small movement which the sun might make in increasing its north or south declination, after its meridian transit on the day previous to its obtaining its greatest declination, we obtain the two extreme meridian altitudes of the sun. Half the difference of these two altitudes gives the meridian altitudes of the equinoctial, which is the value of the colatitude; and hence we obtain the latitude.

Having obtained the latitude, we at once know

the altitude of the pole of the heavens, and hence the distance of our zenith from the pole of the heavens; whilst there would be great difficulty in measuring the actual altitude of various stars from an horizon, on account of this horizon either not being defined or being elevated by refraction, or depressed in consequence of the curvature of the earth. No such difficulties present themselves when we make use of the zenith as a zero point. Refraction does not affect the zenith, and we can, by aid of a trough of mercury, so easily test the verticality of a transit instrument that stars' positions are ascertained by their zenith distances at the instant they cross the meridian, and their declinations, or polar distances, at once deduced therefrom.

Hence, although we give the position of stars in a catalogue by means of their north polar and south polar distances, yet their zenith distance is what is really found, and the other items deduced therefrom. Thus it is of the utmost importance to ascertain what changes, if any, take place in our zenith, besides that produced by the rotation of the earth on its axis.

The right ascension of stars is found entirely by means of their meridian transit. The zero from which right ascensions are counted is the meridian passing through the first point of Aries. When this point of the ecliptic is on the meridian, it is 0 hours siderial time; and as the meridian is carried round by the diurnal rotation of the earth, various stars transit this meridian, and their distance in time from the

first meridian is the measure of their right ascension.

Consequently the earth, with its transit, may be aptly compared to a large theodolite, the diurnal rotation being similar to the revolution of the instrument round its parallel plates, and the revolution of the transit in the meridian being the same as the movement of the telescope by means of the vertical arc.

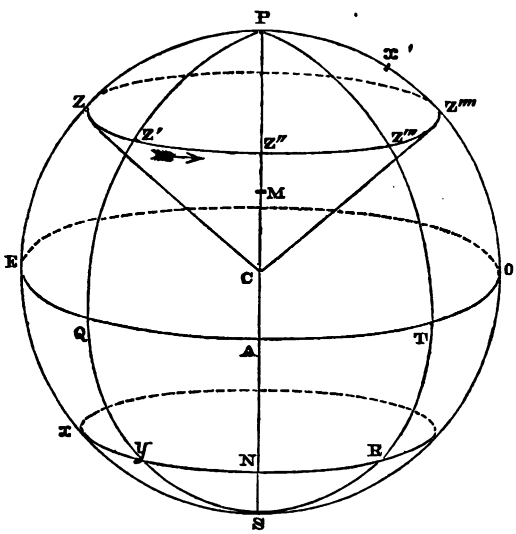
As it is essential that the instrument we use should either remain in perfect adjustment, or we should know exactly what are the changes which do take place, and which must be allowed for, so is it necessary that every change which occurs in our zenith and in our meridian should be known to the greatest certainty; for although we refer in our catalogues to a star's declination and right ascension, yet these items are obtained solely from zenith distances and meridian transits.

Before proceeding in our inquiries relative to more complicated movements, we will first examine the effects of a diurnal rotation on a zenith and a meridian.

Referring to diagram, page 17, let P represent the position of the north pole of the heavens; E Q A T O, the equinoctial; S, the south pole; Z, the zenith of a locality in north latitude 51° 30′. We have then the value of the under-mentioned arcs as follows:

$$E\,Z = 51^\circ\ 30'$$
$$Z\,P = 38^\circ\ 30'$$

Take Z X as 90°, then X will be the point on the southern horizon where the meridian cuts the horizon at right angles, and E X=90°—Z E=38° 30′. Also take Z X′ as 90°, then X′ will be the point on the northern horizon where the meridian cuts the horizon at right angles, P X′=90°—P Z=51° 30′.



Now these angular distances will always remain constant, during any number of rotations of the earth. P X′, the altitude of the pole, will always equal Z E, the latitude; and X E, the meridian altitude of the equator, will always be equal to the colatitude.

The rotation of the earth round the axis P C S of diurnal rotation will cause the zenith Z to pass from Z to Z′, Z″, Z‴, Z⁗, during 12 hours. If we join Z by an imaginary line with C, the centre of the earth,

this line, as it is carried round by the zenith from z to z'''', traces half a right cone, the whole cone being traced out when the zenith has completed its circle in the heavens.

The angle at the vertex of this cone will be equal to twice the colatitude of the locality on earth of which z is the zenith. Thus, for a latitude of 51° 30′, the angle at the vertex of the cone is 77°. At the equator the cone becomes a circle, and at the poles the cone shuts up, as it were, into a straight line, which is the earth's axis.

The number of degrees contained in the circle traced by a zenith, when this circle is seen from the centre of the sphere, are dependent on the latitude of the place; and the number of degrees passed over by the zenith at any time compared to the degrees passed over by the equator may be found from the equation $\cos \mathrm{Z\,E} \times \mathrm{E\,Q} = \mathrm{Z\,Z'}$; applying this equation when the latitude is 51° 30′, it will be found that the circle described by the zenith contains about 224° 6′.

If we found two stars which were 38° 30′ each from the pole, and distant from each other 9° 20′ 12″, we should obtain 15° for the arc of the equinoctial intercepted between the two meridians passing through these stars, therefore the two stars would differ in right ascension 15°, or 1 hour.

When half a rotation of the earth has occurred, and the zenith of 51° 30′ has reached the position z'''', it will be distant from x′ 51° 30′ − 38° 32′ = 12° 58′; that is, the point reached by the zenith of 51° 30′ at

intervals of 12 hours is 12° 58′ from that point which was on the northern horizon 12 hours previously.

If we suppose that the meridian P Z E passes through the first point of Aries, then the meridian P Z″ A S would be a meridian of 90° or 6 hours right ascension, P Z″″ O S a meridian of 12 hours or 180° right ascension, and so on. A star situated at M would have 6 hours' right ascension, whilst its declination would be measured by the arc A M; a star at X′ would have 12 hours' right ascension, whilst its declination would be measured by the arc O X′.

When a star catalogue has once been formed, and the correct position of all stars has been given, this catalogue would hold good for all time, if the only movements of the earth were its rotation on its axis, and its revolution round the sun. The discovery of the advance of the equinoctial point, however, will at once explain why the zero meridian, from which longitudes or right ascensions are counted, itself shifts its position, and thus a change is produced in the longitudes and right ascensions of every celestial body, which requires a correction to be made, to enable an observer to make use of an old catalogue of stars, for the purpose of finding the position of stars at his own time.

This shifting of the equinoctial point has been attributed to the earth's axis tracing a circle in the heavens round the pole of the ecliptic as a centre, always maintaining from it the same distance, as of course a circumference must from its centre, and

producing, as regards the stars, exactly the same effects as if the sphere of the heavens to which the stars appear to be attached itself rotated.* Hence each star in a long period of time would appear to describe a circle round the pole of the ecliptic as a centre, but always maintaining the same distance from this centre, and consequently never altering its latitude or colatitude. This uniform movement of every star would cause a uniform increase in each star's longitude, the rate of this uniform increase being obtained entirely from comparing observations made at different dates.

From an investigation of this movement (which is a very slow one, being performed at the rate of about 1° only in 72 years) it would appear that, as the only item which varies is the longitude, and as the rate of this variation is known, it could of course be calculated for any date. A catalogue of stars for any past date could at once be adjusted for a future date by either adding the proper amount to the longitude of every star, or substituting the proper longitude in the equations already given, and by which the declination and right ascension have been obtained from the latitude and longitude.

It would be a fact that we could thus correct a star catalogue framed for one date into a useful one for another date, if we did accurately know what the movement was which produced the changes in star longitudes; but it is found that practically this can-

* Herschel's *Outlines of Astronomy*, article 313.

not be done; that, from some cause, the theoretical and real position of stars for a future date do not agree; that nearly every star in the heavens is found after a number of years not to occupy that position which it ought to occupy according to theoretical conclusions. Hence it has been assumed that the only possible explanation of this discordance is, that every star has a movement of its own which causes it to wander from that position where it ought theoretically to be located.

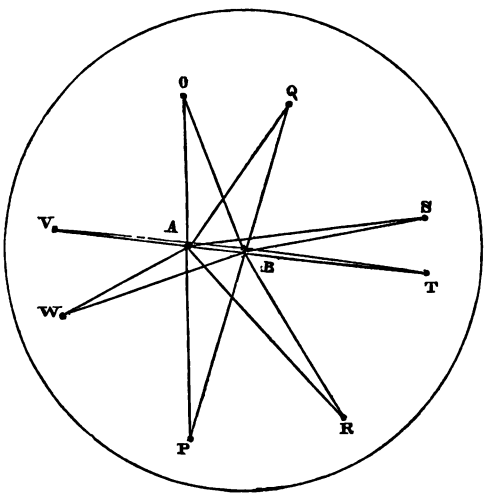
This discordance in the actual and theoretical position of the stars has been given the name of the 'proper motion of the fixed stars;' and to ascertain the amount and direction of this proper motion has been one of the great objects of modern observers of every nation. Telescopes of the best and most expensive kind have been employed; observers by hundreds have devoted nights and years of observation in order to tabulate the results; and at length it has been supposed that the key to the problem had been discovered, and that all these proper motions were due to the whole solar system having a vastly rapid movement in space, towards that part of the heavens which can be localised when we state it had in 1790 a AR* of about 260°, and a north polar distance of about 55° 37′.

The reason why this conclusion has been arrived at is the following:

Suppose O, Q, S, T, R, P, W, V, various stars whose

* AR stands for Right Ascension.

distance from the earth is finite, and suppose the solar system and the earth to be at A. The difference in right ascension of the stars Q and R would be measured by the angle Q A R, and the difference in right ascension of the stars S and T by the angle S A T, and so on. Now if the earth and the solar system moved on to B in the direction A B, the stars Q and R would have a difference in AR represented by the



angle Q B R, whilst the stars S and T would have a difference in AR represented by the angle S B T. Thus the stars in the direction of A B produced would increase their relative right ascensions, whilst those in the direction of B A produced ought to decrease their relative right ascensions. The relative change in the right ascension of the stars at Q and R would be slightly greater than that in the right ascension of

the stars at S and T; also the stars in the direction of S and T would appear to be separating whilst those in the direction of V and W would appear to be closing up.

From an examination of the supposed changes in the AR of various stars it has also been assumed that some stars, being much nearer the solar system than are others, will change their relative right ascensions much more than will other stars; and thus, although a certain law ought to be manifested relative to nearly all stars, yet the detail results will vary in many respects as regards even stars which appear close together in the heavens.

In addition to the general effects resulting from the change in direction of the earth's axis, and termed the '*precession*,' there are two other minor changes in the position of the stars, one of these is called the Nutation, the other Aberration.

The effect known as nutation produces a sort of wavy movement of the pole of the heavens, which, if it occurred alone, would cause the earth's axis to trace a small ellipse in the heavens in about 18·6 years, the longer axis of this ellipse is $18''\cdot5$, the shorter $13''\cdot74$.

In consequence of the pole of the heavens moving steadily in a curve at the rate of about $20''$ annually, this ellipse cannot be completed, so that a wavy motion is given to the pole, and the movement is compensated in about 18·6 years.

The effect known as aberration, and which has

been attributed to the velocity of light, causes an apparent displacement in the stars according as the earth is at one point or at another point in its orbit. Aberration is compensated (as we may term it) every year, so that when the earth is situated relatively to the sun and the stars in the same manner aberration is a constant.

In all future investigations in this work we will not refer to these minor changes resulting from nutation and aberration; but we shall deal alone with the effects of general precession, and assume that we adopt a process of equating observations for nutation and aberration.

We have now described four supposed causes for the changes in position of the fixed stars; there is yet one more to which we must refer. From comparisons made with reference to the obliquity of the ecliptic at various dates, it has been found that this obliquity is gradually decreasing. The cause of this decrease has been supposed to be due to a shifting of the plane of the ecliptic in such a manner as to cause the planes of the ecliptic and equinoctial to come more nearly into coincidence. This change, it has been supposed, does not produce any marked effect upon the right ascensions or declinations of stars; for it has been assumed that notwithstanding the known decrease in the obliquity, yet the pole of the ecliptic still remained the centre of the circle traced by the earth's axis in the heavens. The radius of this circle has been considered a constant in spite of the change

in angular distance of the poles of the ecliptic and equinoctial, and this constant has been assumed to be 23° 28′. If the radius of this circle has not been considered a constant, one of two assumptions only are left us. First, that the pole of the ecliptic always acts as the centre of the circle traced by the earth's axis, and consequently that the pole of the heavens always moves exactly at right angles to the arc joining the pole of the heavens to the pole of the ecliptic. If such a condition prevailed, the course traced by the pole of the heavens would not be any part of a circle, but it would approach more nearly to a spiral, the deviation from a circle being dependent in amount on the extent to which the pole of the ecliptic moves in any given interval of time. Again, if the pole of the heavens were supposed to trace a true circle on the sphere of the heavens, then this circle must have a fixed centre, and the pole of the ecliptic must vary its angular distance from this centre.

If the latter supposition be true, then no change in the right ascension of the stars would occur other than would result if the whole heavens slowly revolved round this centre of the polar circle.

If, however, the former supposition be true, then the changes produced in star positions would be the same as if the stars slowly revolved round the pole of the ecliptic as a centre, the radius of this spiral being variable to the amount moved over by the pole of the ecliptic.

Before proceeding to investigate closely the various

results due to the real motion of the earth's axis, we will refer to the present orthodox belief relative to the changes in stars' position.

First, the changes found in the declination and right ascension of stars are attributed to the pole of the heavens tracing a circle round the pole of the ecliptic as a centre.

Secondly, the discordances between the actual and theoretical position of stars are supposed to be due to the movement of the solar system itself and to the actual movement of the stars.

Thirdly, the supposed changes in the latitudes of stars are attributed to the change in position of the plane of the ecliptic, but how or in what manner this plane changes is not known, nor how this change is effected by the assumed travelling of the solar system in space is not thoroughly known.

Fourthly, the measurement of time being dependent for accuracy on an exact knowledge of the position of the centre of the circle traced by the earth's axis, it is assumed that we have an exact standard measure of time by taking the successive transits of the pole of the ecliptic as a measure of a mean sidereal day, and of the time occupied by the earth in one rotation.

CHAPTER II.

VIEWS HITHERTO ACCEPTED AS REGARDS THE COURSE OF THE EARTH'S AXIS IN THE SPHERE OF THE HEAVENS.

IN our late work, *The Cause, Date, and Duration of the last Glacial Epoch*, we stated that additional evidence of an astronomical nature would be given in the present work, to prove that the course of the pole of the heavens was not such as had hitherto been supposed, but was of such a nature as to fully explain not only the facts of astronomy, but also those of geology, as evidenced in the glacial epoch. In this and the subsequent chapter the present accepted theories will be examined, and the evidence pointed out on which they rest, and also the reasons why a movement of the pole different from that at present accepted as orthodox appears to be that movement which really occurs.

Since the publication of our last work attempts have been made by some astronomers and theorists to assert that there never was a belief that the pole of the ecliptic was the centre of the circle traced by the earth's axis. In our last work we demonstrated

the impossibility of the pole of the ecliptic being the centre of a circle, from the circumference of which circle this supposed centre, it was agreed, varied its distance. To continue, therefore, to assert that the pole of the ecliptic was the centre, was absurd; but to endeavour to deny that such a theory had been accepted as sound, during upwards of two hundred years, is, we maintain, inexcusable; and we believe every impartial reader must acknowledge that it would have been more to the purpose for theorists to have owned to their former mistakes rather than endeavour to explain them away, and to treat them as mere popular methods of speaking of supposed facts.

That there should be no mistake about the former views held by those astronomers who were looked upon as unquestionable authorities, we quote from their works their remarks relative to this important problem.

Upon examining the writings of the acknowledged authorities on astronomical subjects, we find the following as the explanation given of the course traced by the earth's axis in the heavens:

'It is found, then, that in virtue of the uniform part of the pole, it describes a circle in the heavens round the pole of the ecliptic as a centre, keeping constantly at the same distance (23° 28′) from it in a direction from east to west, and with such a velocity that the annual angle described by it in this its imaginary orbit is 50″·10; so that the whole circle

would be described in the above-mentioned period of 25,868 years.'*

There can be no mistake as to what was meant by the above statement. Sir John Herschel but records the theory that was accepted by the school of astronomers of which he was one of the acknowledged heads. He states that the pole of the ecliptic *is* the centre of the circle traced by the earth's axis, and never varies its distance of 23° 28′ from this centre. In order to make this description even more definite, the same distinguished writer adds that the annual rate is 50″·10, and therefore the whole circle will be completed in 25,868 years.

If we find by proportion how many years would be required to complete 360°, when in one year 50″·1 is completed, we obtain 25,868 years.

Nothing can be more clearly defined than is this supposed movement of the pole of the heavens round the pole of the ecliptic as a centre, from which centre the pole is supposed never to vary its distance. Two separate statements are made, each defining the movement in unmistakable terms, and so there is no possibility of misunderstanding that which was meant to be taught.

When it was supposed that the obliquity of the ecliptic was a constant quantity, the above description of the movement of the pole presented no geometrical impossibility; but it is useless to deny that when a decrease in the obliquity of the ecliptic was

* From Herschel's *Outlines of Astronomy*, article 316.

known to occur, then the above description ceased to be correct, because a decrease in the obliquity is the same thing as a decrease in the angular distance of the two poles, viz. that of the ecliptic and of the heavens, consequently these two poles did not always maintain the same angular distance of 23° 28′ from each other; therefore the one pole was not the centre of the circle traced by the other pole, and as it was not this centre we must search for a centre elsewhere, provided the course of the pole of the heavens is part of a circle. When, then, we find that it is and has been for some years an acknowledged fact that there is a decrease in the obliquity of the ecliptic of about 48″ per century, but that the statement relative to the pole of the ecliptic being the *centre* of the *circle* traced by the earth's axis is still repeated, and the assertion still made that these two poles never vary their distance from each other, it is evident that a great oversight has been committed by such theorists, and to attempt to defend such oversight is useless when the defenders are in opposition to any person versed in even the mere elements of geometry.

That the opinion expressed in *Outlines of Astronomy*, was that held by the majority of astronomers is shown by the fact that the statement therein has not only never been contradicted, but has been repeated or copied by all subsequent writers—a fact that proves that the doctrine was accepted on faith, and without investigation; for it does not require any very pro-

found knowledge to convince us that the centre of a circle does not vary its distance from the circumference.

We will now refer to the explanation of the same problem given by the Astronomer Royal in his able *Lectures on Astronomy.* Quoting from page 145 of his work, we find that two lines, viz. C P and C p, represent the two directions of the earth's axis produced by the polar movement, whilst C Q represents the direction of the axis of the ecliptic. This distinguished authority then states:

'Consequently the inclination of C P or C p to C Q is always the same. Therefore we may represent the motion of the earth's axis by saying, that it turns slowly round an axis perpendicular to the ecliptic, but keeping the same general inclination to it in the direction in which the hands of a watch turn (as viewed on the outside of a celestial globe), or in what astronomers call a retrograde direction. Now the pole of the heavens is the point in the heavens to which the earth's axis is directed; and, therefore, that pole is not absolutely invariable, but turns slowly in a circle, in a retrograde direction, round another point to which the line C Q is directed. The latter point is called the pole of the ecliptic.'

This explanation is identically the same as that given in *Outlines of Astronomy;* it claims that the axis of the earth, C P or C p, maintains always the same inclination to the axis, as we may term it, of the ecliptic (C Q). It claims that the pole of the

heavens turns slowly in a *circle* round another point as a centre, which other point is the pole of the ecliptic.

That no misunderstanding may arise relative to the view taken of this movement, we find at page 147 of the same work a repetition of the statement already quoted from *Outlines of Astronomy*, viz. that 'the annual motion of the first point of Aries is about 50″ per annum; it will require about 26,000 years to perform the entire revolution.'

Every portion of this statement might possibly be true, if there were no variation in the obliquity of the ecliptic; but when it is known, and has been long admitted, that there is a variation in the obliquity of about 48″ per century, the above speculations relative to the movement of the pole of the heavens are simply impossible.

In such a work as the present it is necessary to call attention to the above facts, and to demonstrate that the present accepted belief, relative to the movement of the pole of the heavens, is not only geometrically unsound, but that the problem has not received that searching investigation by the astronomical authorities of the past which the importance of the results demanded. That we are compelled to do so is unfortunate, for we at once raise against us the opposition of those authorities who have subscribed to the former erroneous theory; and before it can be granted that our problem has any merit, it must be admitted that former theorists were in error, an admis-

sion somewhat unlikely on the part of the theorists themselves. When, then, it happens that the number of persons capable of judging independently of an original and difficult problem in geometrical astronomy, are to the number who are the mere blind followers of 'authorities in science' as about one to ten thousand, we find ourselves in a considerable minority. Our difficulties, however, are increased when bold attempts are made to deny that erroneous theories were accepted previous to the publication of our former work, although the writings of such theorists clearly show that the theories were deemed unquestionable. That there should be no doubt whatever as to the supposed movement of the pole of the heavens, believed in by the authorities of the present day, we again refer to Sir J. Herschel's *Outlines of Astronomy*, Article 313, wherein he states the following:

'The immediate uranographical effect of the precession of the equinoxes is to produce a uniform increase of longitude in all the heavenly bodies, whether fixed or erratic, . . . as if the whole heavens had a slow rotation round the poles of the ecliptic, in the long period (25,868 years) above mentioned.'

Nothing can be more distinct than is this description. Unless the pole of the ecliptic was, and remained, the centre of the circle traced by the earth's axis, the increase of longitude in celestial bodies would not be uniform; this additional definition therefore must leave no doubt as to the supposition above referred to.

Let us, however, quote another standard work, viz. *The Elements of the Theory of Astronomy*, by J. Hymers, B.D., Article 72:

'Also, if catalogues of the stars constructed as explained in Article 28 for different epochs be compared, it is found that the places of the stars are differently described in them. The right ascensions and declinations are variously altered; the longitudes are all increased by the same quantity, whilst the latitudes remain the same; and the different stars retain the same positions with respect to each other, the same effect as would be produced if all the stars described circles in planes parallel to the ecliptic, with a common slow motion from west to east about an axis passing through the poles of the ecliptic.'

Another writer, whose work was published in 1819, refers to this problem. In *Elements of Astronomy*, by Dr. Brinkley, page 70, it is stated:

'The pole of the celestial equator appears to move with a slow and nearly uniform motion, in a lesser circle round the pole of the ecliptic. . . . The period of the revolution of the celestial equinoctial pole about the pole of the ecliptic is nearly 26,000 years.'

In another work, viz. Lardner's *Handbook of Astronomy*, the present accepted theory is explained relative to the course traced by the earth's axis, so that no mistake can be made as to what has hitherto been supposed relative to this theory; and, consequently, the attempt now to escape from the impossible theory of the circle traced by the pole of the heavens vary-

ing its distance from the centre is, we maintain, a failure. 'If upon any star map a circle be traced round the pole of the ecliptic at a distance from it of 23°·5, such circle will pass through all positions which the pole of the equator will have in time to come, or has had in time past. . . . To determine the period in which the equinoctial points moving backwards constantly at this mean rate would make a complete revolution of the ecliptic, it is only necessary to find how often 50″·1 must be repeated to make 360°; or what is the same, to divide the number of seconds in 360° by 50″·1, which gives 25,868 years.' Lardner's *Handbook of Astronomy*.

From the above quotations it is evident that the idea universally held by astronomers was, that the pole of the heavens moved in *a circle* round the pole of the ecliptic *as a centre;* that it was merely necessary to multiply the number of years occupied in producing a precession of 1° by 360, in order to find the time occupied by the equinox in completing an entire revolution; so that, as stated by the writers we have quoted, if it occupied one year to cause a precession of 50″·1, then it would take 25,868 years to cause a precession of 360°.

It is interesting to find how singular a confusion appears to exist in the minds of those persons who either do not understand, or are indisposed to accept, the demonstrations we have brought forward. Some writers, in finding fault with our last work, state that it *is not* believed that there is a decrease in the

obliquity of the ecliptic: a statement which at once shows these writers are unacquainted with the well-known facts of astronomy. They announce that the movement which all persons really believe in is, 'That the pole of the earth describes a circle of radius 23° 28′ round the pole of the ecliptic as a centre, and the whole circle would be described in something over 25,000 years.'

The individuals who put forward such a statement little know how much they have committed themselves; for those persons who are more acquainted with recorded facts know that the decrease in the obliquity is as well known as is the rotation of the earth; and the great aim of those astronomers who value their reputation is to endeavour to prove that they never did believe that the pole of the earth described a circle of radius 23° 28′ round the pole of the ecliptic *as a centre.* It is always interesting to find science in a transition state; when some individuals have accepted a part of a novelty, but cannot take it all in, whilst others remain as they were half a century ago.

The very first assumption connected with this problem cannot be correct. The pole of the ecliptic cannot be the centre of the circle traced by the earth's axis, in consequence of there being a decrease in the angular distance between the supposed centre, the pole of the ecliptic, and the circumference traced by the earth's axis. Consequently the conclusions based on the supposition that the one pole is the

centre of a circle traced by the other pole must be unsound. There is no evidence to prove that the entire revolution of the equinoxes will take place in 25,868 years. To assume that it will do so because in one year a precession of $50''\cdot 1$ occurs exhibits an entire misunderstanding of the real geometrical problems involved in the phenomenon. Whilst, then, it has been possible, by almost ceaseless observation and labour, to predict for a few years in advance the positions of celestial bodies, yet for long periods in advance, or for minute accuracy, it has not been possible to predict where any star ought really to be situated, except by means of empirical rules, which were found to fail after a short trial.

The particular problem on which former astronomers have required information is with reference to the variation in the obliquity of the ecliptic; for on this subject there was considerable obscurity. Former theorists believed that the pole of the ecliptic was always the centre of the circle traced by the earth's axis, and that they could explain the known change in the obliquity by attributing to the ecliptic some movement which caused the two planes of the ecliptic and equator to come more nearly into coincidence. It appears that such speculators omitted to notice that, if the plane of the ecliptic moved in any way, the pole of the ecliptic must move equally, and must therefore vary its distance from the circumference of that circle of which it was the supposed centre. Such an anomaly must exist according to

the theory which has so long been accepted as unquestionable, and it therefore is not out of place to pursue a purely geometrical investigation of this problem in a manner, and with a detail, that has not hitherto been adopted. In the following pages therefore we deal with the practically geometrical portion of the polar movement, omitting all theoretical assumptions, and keeping entirely to observed facts.

Up to the publication of our work, *The Last Glacial Epoch*, astronomers had taught that the pole of the ecliptic was the true centre of a circle traced by the earth's axis. We have shown that this point cannot be the centre, and we now give additional evidence to demonstrate where the true centre is located, and what the course of the pole really is.

CHAPTER III.

THE COURSE OF THE POLE OF THE HEAVENS, AS DEMONSTRATED BY OBSERVATIONS OF TWO THOUSAND YEARS.

The course traced by the pole of the heavens may be ascertained approximately by means of the decrease in the polar distance of certain stars. Taking the observations of the past 118 years for comparison, and comparing the declinations of stars given in the *Nautical Almanac*, 1873, with those given in Bradley's Catalogue of 1755, the following changes are shown, viz. that the pole of the heavens has decreased its distance from

γ Pegasi . .	39′ 23·76″	since 1755.
α Andromedæ	39 8·3	„
γ Cephei . .	39 26·2	„

Whilst it has increased its distance from

β Leonis . .	39′ 30·0″	since 1755.
γ Ursæ Majoris	39 31·9	„
η Virginis . .	39 27·5	„

The three stars first named are in the direction of 0 hour's right ascension, those last named are in the direction of 12 hours' right ascension.

If we take such a star as μ Sagittarii, which has about 18 hours 6 minutes' right ascension, we find

that the pole since 1755 has moved away from it only 26″·8.

In order to endeavour to trace out the exact course of the pole of the heavens, as indicated by the decrease in polar distance of certain stars, we give the following list of the decrease in north polar distance of stars between 1850 and 1755; and it will be seen that during that interval there is evidence to prove that the pole of the heavens has always moved *approximately* towards that point which, for the time being, was the first point of Aries:

STAR'S NAME.	DECREASE IN POLAR DISTANCE.	AR IN 1850. h. m. s.
78 Pegasi	31′ 32·6″	23 36 27
20 Andromedæ	31 33·8	23 38 36
5 Cassiopeæ	31 41·9	23 39 44
2 Ceti	31 45·7	23 56 3
α Andromedæ	31 31·3	0 0 38
β Cassiopeæ	31 27·1	0 1 12
22 Andromedæ	31 44	0 2 32
γ Pegasi	31 43·8	0 5 30
θ Andromedæ	31 42·8	0 9 16

If the stars be fixed objects, and if the pole did move exactly towards that point which for the time being is the first point of Aries, then the star α Andromedæ would indicate the greatest change in polar distance. But such evidence does not exist; for there are other stars which are some degrees from the first point of Aries which indicate greater changes.

A similar fact is indicated when we examine the changes in north polar distance of stars in the opposite part of the heavens, viz. near the meridian of 12

hours' AR; for we find the following as the changes in north polar distance of stars between 1755 and 1850:

STAR'S NAME.	INCREASE IN N. POLAR DISTANCE.	AR IN 1850. h. m. s.
β Leonis	31′ 47·3″ . . .	11 41 4
β Virginis	32 6·1 . . .	11 42 52
γ Ursæ Majoris . .	31 41·2 . . .	11 45 55
$\varkappa$ Virginis	31 48·1 . . .	11 53 11
o Virginis	31 43·3 . . .	11 57 34
α Corvi	31 47·6 . . .	12 0 41
ϵ Corvi	31 42·8 . . .	12 2 25
δ Ursæ Majoris . .	31 48·3 . . .	12 7 58
γ Corvi	31 41·7 . . .	12 8 5
2 Canum Venaticum	31 48·9 . . .	12 8 35
7 Coma Berencis .	31 46 . . .	12 8 44
c Virginis	31 51·7 . . .	12 12 43

Which gives about 20″·1 for the annual rate of change.* From these changes it is manifest that we have no evidence to demonstrate and prove that the

* In our former work we accepted as correct that the change in direction of the earth's axis amounted to 20″·05 per annum. We consequently made our calculations on the assumption that this was the annual rate of change, and it led us to the date 2298 A.D. as the period at which the pole of the ecliptic, the pole of the heavens, and the centre of polar motion would be on the same great circle of the sphere. Upon making a more searching investigation into the change in north polar distance of certain stars, in the change in stellar longitudes, and in other variations due to, and produced by, the change in direction of the earth's axis, we have evidence that the change in direction of the earth's axis amounts to at least 20″·158 per annum; and the date consequently at which the pole of the heavens, the pole of the ecliptic, and the centre of polar motion will be on the same great circle of the heavens will be 2295½ A.D., or two years and a half earlier than the date announced in our former work. The difference is trifling, but the recorded observations of the past demonstrate it to be so.

The differences produced in the various values of the obliquity will

pole of the heavens has moved during the past 100 years *exactly* towards that point which was for the time being the first point of Aries. If the pole had so moved, and the stars are fixed, then the star α or ε Corvi would indicate a greater increase in north polar distance than would either π Virginis or c Virginis; and if we grant that the stars themselves move, then we cannot claim as evidence of a polar movement increases or decreases in distances that may be due to an actual movement in the stars themselves; consequently, we require some additional evidence before we can define the true course traced by the earth's axis in the heavens.

The Pole-star α Ursæ Minoris has at all times been considered an important star by astronomers, and the distance of this star from the pole has been carefully noted from very early ages. We find that whilst many stars mentioned in Bradley's Catalogue of 1755 were observed only five or six times, the

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vary scarcely a second from those given in our last work; but these differences, in almost every case, cause our calculated obliquity to come more nearly into coincidence with the obliquity found by observation at the various dates, and the principal fact is not altered in the least, viz. that the pole of the heavens appears to trace a circle round the pole of the ecliptic, not as a centre, but round a point as a centre 6° from the pole of the ecliptic.

The principal reason for our concluding that the pole does change its position to a greater amount than $20''\cdot05$ per annum is the fact that several stars decrease or increase their polar distance as much as $20''\cdot1$ per annum; and when the proper allowance is made for the pole of the heavens moving in a *small* circle of the sphere, we obtain $20''\cdot158$ as the most approximate value per annum for this change in direction of the earth's axis, and it is taken as such in the calculations in this work.

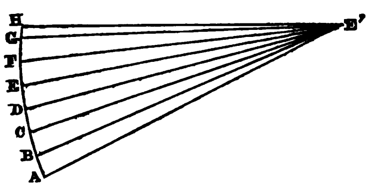
Pole-star was observed no less than 217 times. In later catalogues every attention has been given to the position of this star, and so it may be made use of as a check as regards the real course of the pole of the heavens; for the distances at various dates of the Pole-star from the pole have been carefully recorded. Thus we find that the Pole-star was distant from the pole

		Approximate rate per annum.
1873	1° 22′ 4·3″	19·180
1860	1 26 13·65	19·107
1850	1 29 24·72	19·338
1845	1 31 1·41	
1819	1 39 25·05	19·370
1755	2 0 18·9	19·691

We have here a variable set of distances, which will prevent any correct result being obtained by any empirical rule, unless depending solely on recorded observations of the past. We have therefore data on which to trace out the course of the pole; for we have a series of points on the surface of a sphere; and the line connecting these points either has or has not a definite character—it is either an irregular line, or it is part of some circle, some ellipse, or some known curve. If the line joining these points be what we may term small arcs of circles, each circle having a different radius, the line will not be any defined curve; but if we can find that the line has a defined character, that it is part of a circle the centre and radius of which can be defined, we at once make a great advance in our knowledge of the true course traced by the earth's axis.

It has hitherto been stated by certain astronomers that the movement of the pole of the heavens and the supposed movement of the pole of the ecliptic were of so complicated a nature as to render it impossible to calculate what the obliquity of the ecliptic should be for any long period in advance, by any process other than an empirical rule of subtracting 48″ per century from the known value at any date, in order to find what the obliquity should be for a century in advance. This statement, it will be seen, has been made in consequence of the true geometrical course of the pole not having been known, and it will be demonstrated that the value of the obliquity can be calculated when we know the true course of the pole.

We will now refer to the pole of the heavens and to the pole of the ecliptic, and note the changes in the distances that have occurred during about 230 years between these two points. Thus, let E be the pole of the ecliptic:



A, the pole of the heavens at the date				1590
B,	„	„	„	1690
C,	„	„	„	1743
D,	„	„	„	1795
E,	„	„	„	1815
F,	„	„	„	1824
G,	„	„	„	1840
H,	„	„	„	1871

Now, the values of E′ A, E′ B, &c. are known and have been recorded, the length of the arcs A B, B C, &c. are known, because the rate at which the pole moves is known. To find the value of B E when we know

that of A E′ and A B is not possible, unless we know the value of the angle E′ A B, or A B E′, or B E′ A, and to know these angles requires an *exact* knowledge of the course traced by the pole in moving from A to B. The value of the arcs E A, E B, &c., for various dates, are as follows, which arcs are in reality the obliquity of the ecliptic at the dates named, viz.

DATE.	OBLIQUITY.			RECORDED BY
1590 . . .	23°	29′	52″ . . .	Tycho Brahè.
1690 . . .	23	28	48 . . .	Flamstead.
1743 . . .	23	28	26 . . .	Cassini de Thury.
1795 . . .	23	27	57·6 . . .	Maskelyne.
1815 . . .	23	27	47·46 . . .	Bessel.
1825 . . .	23	27	44 . . .	Dr. Pearson.
1840 . . .	23	27	36·5 . . .	Professor Airy.
1850 . . .	23	27	31·95 . . .	*Nautical Almanac.*
1860 . . .	23	27	27·38 . . .	,, ,,
1870 . . .	23	27	22·2 . . .	,, ,,

Between 1590 and 1690 it appears there was a decrease in the angular distance of the two poles of 1′ 4″, equal to 64″ per century, or 0″·64 per year.

Between 1795 and 1690 there was a decrease of 50″·4, equal to 0″·53 per year. Between 1840 and 1850 there is a decrease of 4″·55, equal to 0″·455 per year. Thus, from observation, we know that *the rate* of decrease of the obliquity is a decreasing rate.

From these recorded observations we claim, that the course of the pole of the heavens relative to the pole of the ecliptic can be defined with minute accuracy; that the curve indicated by the recorded observations above has a defined character; that it is not, as has been hitherto assumed and supposed,

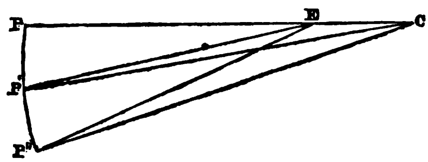
part of a circle, having for its centre the pole of the ecliptic, or a vague curve whose character could not be known, and whose distance from the pole of the ecliptic could not be predicted except for very short periods, and that only by aid of a mere addition or subtraction of certain values—a process which fails for long periods; but the value of the obliquity can be calculated by means of the most severe of all tests, viz. a geometrical investigation.

The curve traced by the pole of the heavens relative to the pole of the ecliptic is along the circumference of a circle, having for its centre a point located as follows:

Let E be the pole of the ecliptic, P the pole of the heavens at the date 2295·5 A.D., and C a point 6° from the pole of the ecliptic and 29° 25′ 47″ from P, then C is the centre of the circle traced by the pole of the heavens.

The statement that the pole of the ecliptic is the centre of the circle traced by the earth's axis does not appear to be based on any evidence, or on any observed facts, or even to be capable of geometrical demonstration; it is a theory only. That the course traced by the earth's axis is along the circumference of a circle having for its centre the point named in the last paragraph, is capable of demonstration and proof, and is not a mere theory, but an actual fact, which may be proved as follows:

The points C E P P′ P″ we take as points on the surface of the same sphere; E represents the pole of the ecliptic, C the centre of the arc, P P″; P″ P′ P the course traced by the pole of the heavens; P repre-



sents the position of the pole of the heavens at the date, 2295·5 A.D. The radius P C is 29° 25′ 47″, and E C is 6°; consequently, P E is 23° 25′ 47″.

If these items are correct, and we have correctly localised the position of C, the centre of the arc P P″, the curve P P″ becomes a known curve; it is known to be a part of the circumference of a circle, the centre of which is C; and if it be such a curve, *then, and then only*, can we calculate the angular distances of P′ E and P″ E, which distances are the obliquity of the ecliptic at various dates.

The importance of this problem and the severity of this test cannot be overrated; we are not dealing here with any theoretical speculation similar to that now popularly supposed to be the movement of the pole, but we have brought this problem to the rigid test of geometry. We have resolved the problem into a simple one, viz. if the curve P P″ is such as to fulfil the conditions shown by the recorded changes in the obliquity during the past, this curve can have but one character, viz. that given above, and it is

impossible that it can be an irregular curve, having for its centre a movable point.

We can now calculate the values of P′ E, P″ E, &c. from the data which we have given above.

From the best recorded observations of the rate at which the polar distance of stars has changed, it is found that the pole moves about 20″·158 per annum. If, then, we wish to find the length of the arc P P″—that is, the angular distance between the position of the pole at the date 2295·5 A.D., and at another date when the pole was at P′, say at 1870 A.D.—we must multiply 425·5 years, the difference between these two dates, by 20″·158, when we obtain 8577″·2 as the value of the arc P P″.

We also have the value of the arc E C $= 6°$ of C P $=$ C P′ $= 29° \; 25' \; 47''$. We can therefore find the value of the angle P C P′; because, knowing P P′ and P C and P′ C, the angle at C is $=$

$$\frac{\mathrm{P\,P'}}{\sin.\ 29° \; 25' \; 47''}$$

When we have found the angle at C, we have in the spherical triangle E C P′ two sides, viz. E C and C P′, and the included angle at C to find the third side E P′, which third side E P′ is the angular distance between the pole of the heavens and the pole of the ecliptic, at the date when the pole of the heavens is at P′.

We may repeat this calculation *independently* for every date for which we require the obliquity; the only variation in the calculation being that the value of the arc P P′ will vary for different dates.

For example, if we wished to calculate the value of the arc P P′ for, say 1690, we should subtract 1690 from 2295·5, and multiply the remainder by 20″·158, and then proceed as before.

Arranging this formula in an algebraical form, we have the following:

$$\{2295{\cdot}5-T\}20''{\cdot}158=\alpha$$

$$\frac{\alpha}{\sin.\ 29^\circ\ 25'\ 47''}=C$$

$$\cos.\ C\ \tan.\ 6^\circ=\tan.\ B$$

$$\cos.\ O=\cos.6^\circ\left\{\frac{\cos.\ 29^\circ\ 25'\ 47''-B}{\cos.\ B.}\right\}$$

T represents the date for which the obliquity is required; O represents the obliquity at that date.

This calculation is perhaps the most rigid geometrical investigation that has ever been applied to an astronomical problem. We have localised the centre and radius of a circle; we have localised a point in the heavens at which we state the pole of the heavens will be situated 420 odd years in advance. Then, from the data thus given, we proceed to calculate what will be the mean obliquity of the ecliptic for any year, for a thousand years in the past, for the present date, and for the future.

If the data on which these calculations rest are true, the results ought to agree with recorded observations. If the data be not true, it is impossible that the calculations can agree with recorded observations. Thus we have no longer to deal with theories, but we come to a fact. Is the curve given above that which the earth's axis really traces? If

it is, then the recorded variation in the obliquity can be calculated; if it be not this curve, then calculations based on the supposition that it is will not agree with recorded observations.

Adopting the calculation given above, we find that not only do the results obtained by our formula agree with recorded observations, but they agree with minute accuracy, as will be seen by a comparison of the following:

DATE.	OBLIQUITY BY OBSERVATION.	BY CALCULATION.
1590	23° 29′ 52″	23° 29′ 54″
1690	23 28 48	23 28 48
1743	23 28 26	23 28 27
1795	23 27 57·6	23 27 58
1815	23 27 47·46	23 27 47·5
1825	23 27 44	23 27 44
1840	23 27 36·5	23 27 36·5
1850	23 27 31·95	23 27 31·4
1860	23 27 27·38	23 27 27

It is here demonstrated, that during 230 years we can calculate what the obliquity was to within one second; that is to say, the actual curve traced by the pole of the heavens relative to the pole of the ecliptic during 230 years does not differ one second from the circumference of a circle having a radius of 29° 25′ 47″, and its centre 6° from the pole of the ecliptic. In other words, the curve traced by the pole of the heavens during 230 years *is* part of a circle such as that defined above.

In making this announcement we are opposed by those astronomers of the day who are looked upon as authorities. It is by them claimed that the pole

of the ecliptic itself is the centre of the circle traced by the earth's axis; and although this centre varies its distance from the circumference, yet they suppose it is always the centre of the circle. If such a movement really occurred, it must follow that the course traced by the earth's axis must be an irregular curve; whereas we have proof that the course is part of a circle having for its centre a point 6° from the pole of the ecliptic.

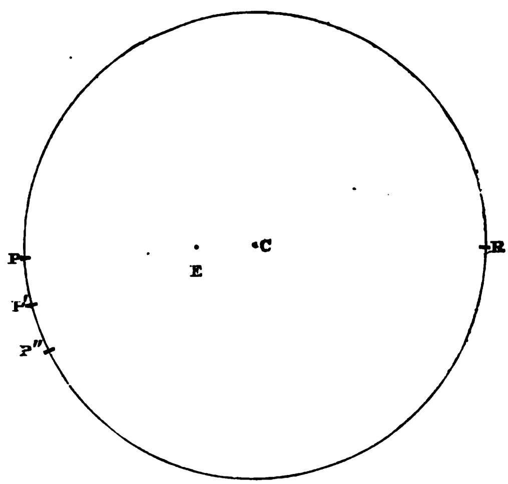
CHAPTER IV.

THE COURSE OF THE POLE OF THE HEAVENS (CONTINUED).

During at least 230 years we can claim to have facts to corroborate the course traced by the pole of the heavens; but it may be urged that the differences are very slight during that period between such a circle as above described and an irregular course such as that supposed to occur by theorists. In order to carry out these differences to their greatest extent, we will trace out the course of the pole relative to the pole of the ecliptic during a long period of time, and call attention to the extreme differences that will occur between that movement, which recorded facts prove to have occurred, and the theoretical course supposed to be that followed by the pole of the heavens.

If the pole of the ecliptic be always the centre of the circle traced by the earth's axis, theorists claim that the variation in the value of the obliquity will be of a very limited amount—not more than 1° 21′ either way—and that therefore astronomy is unable to afford any explanation of the changes in climate on certain parts of the earth known from geological evidence to have occurred.

Following out the course actually traced by the earth's axis during the past 230 years, and keeping rigidly to the facts shown by recorded observations, we can complete the circle indicated as that traced, and we then find the following interesting facts:



C is the point in the heavens indicated as the centre of the circle traced by the pole, E is the pole of the ecliptic, and P P′ P″ R the circle traced by the earth's axis round C, the centre.

On the pole of the heavens reaching the point R, its angular distance from E will be E C + C R = 6° + 29° 25′ 47″ = 35° 25′ 47″.

The angular distance between the two poles R and E is at all times the measure of the obliquity or extent of the arctic circle, which, in the case above, would be 35° 25′ 47″. Therefore, all localities within

the distance of 35° 25′ 47″ of the poles of the earth would be within the arctic circle when the pole was at R, and regions now temperate as regards climate would then have possessed during winter an arctic climate. The date at which the pole would have been at R was about 13,000 B.C.

Upon examining the climatic changes which would occur with an obliquity of 35° and upwards, we find they would be of exactly such a nature as geologists claim to have occurred in what they term the glacial epoch of geology. These climatic changes have hitherto been inexplicable, no theory of them having been found satisfactory.

When, then, we find that the course traced by the earth's axis during the past 230 years is exactly such as to produce, if continued, those climatic changes required to explain the known facts of geology, whilst the same course explains exactly the recorded facts in astronomy, whereas the present accepted theory fails to explain either, it appears that the relative value of these explanations is somewhat different.

From an examination of the reviews or notices of our former work, *The Last Glacial Epoch*, it appears that many critics entirely misunderstood the object of that work. In that book we demonstrated that a curve traced by the pole of the heavens during the past 1800 years would, if continued to 13,000 B.C., cause the climate on earth to be that demonstrated by geology to have existed at the glacial epoch. Whether this curve was continued for all time was a question we left open;

we only claim to demonstrate that, for at least 13,000 years, it is most probable that any secular change which may be at work to alter the character of this curve is so slight as scarcely to be manifested. That which we claim to have demonstrated is, that a curve, proved by geometry to be the true course of the pole of the heavens round the pole of the ecliptic, does give those climatic conditions long shown by geology to have existed during the glacial epoch. Some individuals have selected some item, or, as we may term it, some idiom, in geological evidence, and finding that the problem we have brought forward does not satisfactorily explain that particular item, have considered the problem weak. We have proved that the course of the pole of the heavens is such as to cause an arctic climate in winter down to 54° latitude in each hemisphere at a date 13,000 B.C. That this problem will not explain the rise and fall of land, will not account for earthquakes, or for there being more water in the southern than in the northern hemisphere, is scarcely a sound reason why it does not explain the climate of the earth during the last glacial epoch.

From an examination of the actual recorded facts connected with both astronomy and geology, it appears that, as far as the history of astronomy carries us, the course traced by the pole of the heavens is relative to the pole of the ecliptic, a circle having for its centre a point 6° from the pole of the ecliptic. Having demonstrated this fact, we have solved only one part of a most important problem. We have

shown that, relative to the pole of the ecliptic, the course traced by the earth's axis is the circle traced and defined above. We have the recorded observations of about 1800 years to corroborate this course; and we find that, if continued, we should obtain in about 13,000 years exactly those climatic conditions which the researches of geologists prove did occur at a recent date in the earth's history. We have not, however, yet proved whether the course of the earth round the sun is round a constant plane, whether the orbit of the earth has any secular change similar to that of the moon's orbit in 18⅔ years, or whether the whole solar system appears to have any movement in any particular direction. Also, although we have defined the course of the earth's axis in the heavens, we have not as yet defined what movement of the earth accompanies this so-called conical motion of the axis; *for there are no less than four entirely different movements of a sphere, each of which may occur with exactly the same movement of the axis.* Any one of these movements would produce very different results from another in some details, and it would require long and accurate observations to decide which movement really accompanied the conical motion of the axis known to occur. Scarcely any of these problems have been examined hitherto, or from a geometrical point of view. Theories have too often been substituted for facts, which latter, if not in accordance with accepted theories, have not unfrequently been discarded.

The attention of the reader is now called to the fact that it may be claimed as proved that the pole of the heavens traces, apparently, an arc of a circle round a point 29° 25′ 47″ distant from it; but whether both the centre of the circle, the pole of the ecliptic, and the pole of the heavens have jointly or independently any other motion is a problem which must now be investigated.

The difference between the present accepted theories relative to the movement of the earth's axis and the problem which we have brought forward may be briefly described as follows:

Hitherto it has been supposed that the pole of the ecliptic always was the exact centre of a circle, which circle was traced by the earth's axis. This assumption having been agreed to, it was supposed that the whole period of a revolution of the equinoxes (caused, as it is known, by the movement of the earth's axis) could be calculated by a mere rule-of-three process; so that, if in one year 50″·1 were passed over by the equinox, therefore 360° must take 25,868 years. It was and is supposed that the decrease in the obliquity of the ecliptic known to have continued during the past two thousand years could be due only to an actual movement of the plane of the ecliptic; but the following geometrical laws seem to have been overlooked when this theory was supposed correct:

Granting that there is a decrease in the obliquity, it follows that there is a decrease in the angular distance of the pole of the ecliptic from the pole of the

heavens, or in other words, a decrease in the distance of the assumed centre from the circumference; consequently, we at once encounter an impossibility by such a theory.

In order to avoid this contradiction, a modification of the above assumption has very lately been suggested by certain authorities, viz. that, although the pole of the ecliptic varies its position in the heavens, still the pole of the heavens always moves round it as a centre, or in other words, always traces its annual arc of about 20″·158 at right angles to the arc joining the two poles.

This theory is not the same as that described by any of those authors whose writings we have quoted, and the facts and evidence to prove its truth are wanting. The exact change in the obliquity known to have occurred formerly cannot be calculated by aid of this theory. The exact course of the pole of the heavens cannot be known, because the true course of the pole of the ecliptic is not defined, and therefore the position of the supposed centre of the arc traced by the pole of the heavens is not known. In reality this theory is little better than is the former, except that it does not suppose an impossibility, as in the former case, although it is not based on those results of observation and on those geometrical calculations which should alone be the foundation of a theory.

In the problem which we have brought forward, we first find, from recorded observation, that, relative to the fixed stars, the pole of the heavens moves about

20″·158 per annum somewhere towards that point called the first point of Aries. We find that, relative to the pole of the ecliptic, the pole of the heavens decreases its distance about 0″·45 annually at present; that recorded observations indicated that about two centuries ago it decreased its distance more rapidly, viz. at about 0″·64 per annum. We then analyse the actual curve traced by the pole of the heavens relative to the pole of the ecliptic, and we find it to be actually a circle having for its centre, not the pole of the ecliptic, but a point 6° from that pole. This discovery enables us to obtain, by a sound geometrical calculation, the actual value to a second of the obliquity during many hundred years; to show a cause for the annual decrease in the obliquity being only 0″·45 per annum now and 0″·64 formerly, whilst in more ancient times it was even still more rapid; to show a sound reason why the climate on earth was such as to produce a glacial epoch 15,000 years ago; and to do all this, not by any speculations or theories, based on other speculations or theories, but by merely tracing out the course which the earth's axis has followed during the past three or four hundred years.

Whether or not the pole and plane of the ecliptic vary their position relative to the fixed stars is quite another problem; they may or may not do so. If they do so, the latitudes of stars will be found to vary, and in a regular order; but, even granting that the plane of the ecliptic does vary its position, such variation cannot interfere with the fact that the course of

the pole of the heavens is relative to the pole of the ecliptic, such as we have defined it to be. This course is recorded during many hundred years; and when no recorded observations exist, and we search for evidence other than that of astronomical observation, we find the most undeniable proofs in the testimony of the rocks themselves, in the vast transported boulders, in the evidence of ice-action on that very parallel of latitude where it ought to have occurred in consequence of this movement of the pole.

It is a most important fact that, whilst the arc traced by the pole of the heavens round the centre that we have defined causes an obliquity of $35\frac{1}{2}°$ at its extreme position, and thus brings down the arctic circle to $54\frac{1}{2}°$, and an arctic climate to about 50°, the evidence of geology demonstrates that the great boundary of the glacial climate was confined to a parallel of latitude of about 50°, and did not extend lower, except on mountain ranges, where of course other causes affect the temperature. We have, then, from geometrical astronomy, a proof of the extent of the arctic circle at a remote date, and we have, from geology, evidence that the arctic climate was confined within the same parallel of latitude as that shown by astronomy to have been the boundary of the arctic circle.

An examination of the present accepted theory relative to the causes producing a decrease in the obliquity of the ecliptic has revealed the singular fact that two theories directly contradict each other. By

one theory, a point in the heavens is supposed to be the centre of a circle; by another theory, the same point is supposed to be movable as regards the circumference of the circle. Upon examining two other theories bearing directly on this investigation, we again find a contradiction; for we come across two theories, each supposed to be correct and each supposed to explain one particular effect, yet, on a searching investigation, these theories will be found to contradict each other. The first of these theories is the following:

The plane, and hence the pole, of the ecliptic is supposed to change its position among the fixed stars in such a manner as to cause a decrease in the obliquity of the ecliptic, and consequently a decrease in the angular distance of the pole of the ecliptic from the pole of the heavens. Thus, let P represent the position of the pole of the heavens at any date, E the position of the pole of the ecliptic, P E consequently the obliquity. Let P′ represent the position of the

pole of the heavens at another date, this pole having moved from P to P′ round E as a centre. If E remained fixed, P E would equal P′ E, and no change in the obliquity would have occurred. In order to account for there being a decrease in the distance P′ E, it is supposed that P P′ is traced the same as if E remained the centre, but moved somewhere in the direc-

tion of E′, so as to make E′ P′ less than P E. We must now keep rigidly to this fact; for it is evident that to cause a decrease in the angular distance of P′ from the pole of the ecliptic, the pole of the ecliptic must move somewhere in the direction of P′, and cannot move in the direction of X. When the pole E moves in the direction of P′, it must decrease its distance from all stars in the direction of E P or E P′; that is, it must decrease its distance from, and must therefore be moving towards, that part of the heavens indicated by six hours of right ascension, and must consequently increase its angular distance from all points in the direction of X.

When another paradox required an explanation, it appears that the theory relative to the pole of the ecliptic moving in the direction of six hours' right ascension was forgotten; for a fresh theory was invented which supposed that the pole of the ecliptic was moving in the direction of eighteen hours' right ascension. This theory is the following:

From an examination of the recorded changes in stars' right ascensions and declinations, it has been supposed that the stars had what was called 'proper motions.' From the process adopted to discover these proper motions, it was supposed that the whole solar system, and therefore the earth, the earth's orbit, consequently the ecliptic *and the pole of the ecliptic*, were moving towards the constellation Hercules, which has about eighteen hours' right ascension.

There are consequently two theories now accepted

as true, viz. that the pole of the ecliptic is moving towards six hours' right ascension, and thus explains the decrease in the obliquity, and that the pole of the ecliptic is also, at the same time, moving in the opposite direction, viz. towards eighteen hours' right ascension, and thus explains the supposed proper motions of the fixed stars.

In our work, *The Cause, &c. of the Glacial Epoch*, we called attention to the fact that it seemed improbable that the centre of a circle could vary its distance from its circumference and yet remain the centre, although it had been agreed during nearly two hundred years that it could do so. We now call attention to the fact that it seems equally improbable that the pole of the ecliptic can be moving in two opposite directions at the same time, in spite of the various theorists who at present consider that such a movement satisfactorily explains the problems brought before them.

In the following pages we undertake to demonstrate that the process adopted hitherto to find the so-called proper motion of the fixed stars is unsound in principle, and the results erroneous. Also, we undertake to demonstrate that the discordances found to occur are due almost entirely to the erroneous theories now accepted relative to the movement of the pole of the heavens, and the resulting movement in the earth itself.

CHAPTER V.

THE COURSE OF THE POLE OF THE HEAVENS (CONTINUED).

We have demonstrated in the preceding chapter that the pole of the heavens has traced during the past 230 years a course which, relatively to the pole of the ecliptic, is part of the circumference of a circle having for its centre a point 6° from the pole of the ecliptic. We are disposed to leave the question of the course of the pole during the past 230 years as it stands in the last chapter. We produce the recorded observations, which are facts, and are not disputed by any modern astronomers; we demonstrate what these recorded observations prove by means of a sound geometrical investigation, and this proof cannot be denied. Any person who denies that the course of the pole of the heavens relative to the pole of the ecliptic is such as is demonstrated in our last chapter is simply opposing himself to facts, and is adhering to an old and impossible theory in preference to examining a new and well-demonstrated truth.

Upon passing beyond the date of 230 years, we arrive at a period when astronomical observations were only approximate, and when different observers

at the same date differed several minutes in their observations; so that it is impossible to test our calculations by the recorded observations made many centuries ago. Thus, at the commencement of the Christian era, it appears, no instrument read an angle more correctly than 10′; and to assume that any observer could be certain of any angle to one division of his instrument is a delusion. Hence we find that observers recorded 24° 0′ 0″, 23° 54′, and 23° 51′ as the value of the obliquity at the commencement of the Christian era; and these values may have been 10′ more or 10′ less than those recorded. They all, however, indicate that the course traced by the earth's axis has been such as to cause a decrease in the obliquity of the ecliptic similar to that which would occur if the course traced during 230 years were continued during 1800 years. Whether exactly the same *rate* of movement of the pole has always continued is a question which is open to opinion; but considering the uniformity of all other celestial movements, it is probable that the motion has been uniform.

When we pass beyond the date from which we can glean recorded observations, we come to perhaps as severe a test of the truth or falsity of the problem we have advanced as can be obtained even from our modern observations; for we find that if the present popular theory of the movement of the pole be true, it would follow that there never has been from astronomical causes any change of climate on earth, or due to the sun's change of altitude in the heavens.

If, however, the course traced by the earth's axis during the past 230 years has been traced in past ages, it follows that there was about 15,000 years ago so great a change in the climate as to cause the arctic circle to descend to about 54½° of latitude, and thus to cause an arctic climate to prevail down to that parallel of latitude. An examination of the facts known to geologists shows that this peculiar climate not only prevailed on earth just previous to the present known conditions, but it prevailed only down to about that same parallel of latitude which we find from our geometrical investigation was the limit which ought to have existed.

We have, then, first the recorded observations of modern times, which prove the course of the pole of the heavens to be part of a circle having for its centre a point 6° from the pole of the ecliptic. This discovery enables us to calculate for all past time the exact value of the obliquity, and we find calculation and recorded observation agreeing. By the present accepted theory such a calculation and such results are not possible.

We have, secondly, certain facts in geology proving beyond a doubt that exactly those climatic changes did occur formerly which would have been produced by that course which present observations prove is occurring. By the present accepted theories such climatic changes are found to be 'mysterious,' 'utterly unaccountable,' and totally 'inexplicable by astronomy.'

The question might fairly be left in this position and to time to be decided; but as so important a problem, and one affecting so much the labours and time of astronomers, should if possible be placed even on a firmer basis, we now venture to bring forward another demonstration, which we think will be conceded as the most remarkable problem ever produced in geometrical astronomy.

In the preceding demonstration we determined by calculation the distance of the pole of the heavens from the pole of the ecliptic during 230 years by means of tracing a portion of a circle the centre of which was 6° from the pole of the ecliptic. The decrease in the angular distance of the pole of the heavens from the pole of the ecliptic amounts to only about 0″·45 per annum; so that if an error were made of two years in the value of the obliquity, it would amount only to about 1″ of arc. The test which we will now bring to bear is a more severe one; it is to determine by calculation the distance of the Pole-star from the pole during the past 100 years by means of the curve traced by the pole round the centre which we have defined. If any item in our calculation is incorrect to even a small amount, the resulting error in position of the Pole-star would be very large, and an error of two years in time would produce about 40″ in arc.

The Pole-star is selected for this test for various reasons. First, it has assigned to it no proper motion by modern observers; so that whatever cause

produces those discrepancies assigned to proper motion, these causes do not affect the position of the Pole-star. Secondly, the latitude assigned to the Pole-star by Ptolemy in 140 A.D. is 66°, and the latitude by Hevelius in 1660 is 66° 3′; so that whatever changes or assigned changes occur in the latitude of some stars, the Pole-star seems not greatly affected thereby, as its assigned latitude has varied only 3′ in 1520 years, or less than 10″ per century. Then, again, the observations recorded as being made on the Pole-star are more numerous than are those for any other star.

If we find that the same curve which has enabled us to trace out the course of the pole relative to the pole of the ecliptic also enables us to obtain the polar distance of the Pole-star, we think such an additional proof will be given as will convince all competent geometricians that the curve we have traced out is correct to the minutest degree.

Calculations relative to the Polar Distance of the Pole-star for the past 105 *Years.*

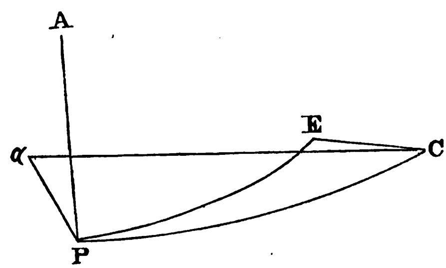
The first calculation which we undertake relative to the above problem is to calculate the value of the angle formed at the pole of the heavens at the date 1873, by two great circles, one passing through the pole of the ecliptic, the other through the centre of polar motion.

Let C represent the centre of polar motion, E the pole of the ecliptic, P the position of the pole of the

heavens on January 1st, 1873. From the data already given we can calculate the distance P E, which is the obliquity, and is 23° 27′ 20″·9. We have, then, three sides of the spherical triangle E P C to find the angles, and we obtain 1° 15′ 44″·6 for the angle E P C.

We will now take the recorded observations of 1873 as regards the right ascension and polar distance of Polaris, and we obtain for these values from the *Nautical Almanac* 1° 22′ 4″·3 for the polar distance, and 18° 4′ 26″ for the right ascension.

In the annexed figure, P represents the pole of the heavens, α the Pole-star, P A the zero meridian



from which right ascensions are counted; E the pole of the ecliptic, C the centre of polar motion.

P α we know, from observation, is 1° 22′ 4″·3; the angle α P A is 18° 4′ 26″; the angle A P E is 90° (as it always must be); the angle E P C has been found by calculation to be 1° 15′ 44″·6; the angle α P C will therefore be the sum of the three angles named, viz. α P A+A P E+E P C=109° 20′ 10″·68.

We have now the spherical triangle α P C, in which

we know the two sides P α and P C, and the included angle at P, to find the third side C α and the angle at C; that is, P α=1° 22′ 4″·3; P C=29° 25′ 47″; and the angle α P C=109° 20′ 10″·68. From these data we calculate the value of C α, which we find to be 29° 54′ 29″, and the angle α C P=2° 35′ 21″·4.

We have now what we will term 'constants' for Polaris, 1873; that is, we have the distance of Polaris from C, the centre of polar motion at that date; and this value ought to be constant, provided the centre of polar motion does not alter its position, and the star does not alter its position. We have also the radius P C of polar motion a constant, viz. 29° 25′ 47″; the angle α C P we know the value of; and having found that the pole of the heavens traces its circular course round C at the rate of 20″·158 per annum, we find that, due to this movement, the angle α C P decreases annually at the rate of

$$\frac{20''\cdot158}{\sin.\ 29^\circ\ 25'\ 47''}=41''\cdot03$$

So that at the date 2100 the right ascension of Polaris will be 6 hours.

If the reader will carefully examine the above calculations, he will find that we can calculate the value of the angle α C P for any date, from the one recorded observation of 1873, and we shall have therefore for any date two sides of a spherical triangle, viz. C α and C P, and the included angle α C P, to find the third side P α, which third side is the polar distance of Polaris for any date.

In this calculation we adopt quite another process from that by which we obtained the obliquity with such accuracy, and we employ other data, although we keep to exactly the same curve traced by the earth's axis. Instead, however, of finding the distances of the pole of the heavens from the pole of the ecliptic, we now endeavour to find the distances of the pole of the heavens from the Pole-star.

Starting from our one observation, and the calculations derived from 1873, we have in the spherical

triangle α P′ C, P′ α, the polar distance of Polaris at 1873. The side C α=29° 54′ 29″; P′ C=29° 25′ 47″; and the angle α C P′=2° 35′ 21″·4.

If the course traced by the pole be such as we have stated, P, the position of the pole at any date in the past, will cause P C to be equal to P′ C; whilst the angle P C P′ will vary to the amount of 41″·03 annually, in consequence of the movement of the pole around C as a centre. The angle α C P′ being 2° 35′ 21″·4 for 1873, the angle α C P would have been 2° 35′ 21″·4+410″·3 at 1863, and so on. We can, then, always find the value of this angle for any date in the past, and we have therefore two sides, C α and C P, and the included angle at C, to find P α, the third side.

If it is found that by this calculation we can approximate even to the polar distances of the Pole-star, we may be tolerably certain that the course which we have defined as that followed by the pole of the heavens is nearly correct; for we make no use of the obliquity of the ecliptic or pole of the ecliptic in this investigation, but refer to three points only, viz. to the Pole-star, the centre of polar motion, and the pole of the heavens.

In all these calculations we calculate the mean position of the Pole-star for the 1st of January of the year named, and as given in the *Nautical Almanac* for each year, with which records we will compare our calculations.

The first calculation we will make will be for the year 1860. For this year the angle α C P′ must be increased by $41''\cdot03 \times 13 = 8'\ 53''\cdot4$; therefore the angle α C P $= 2°\ 44'\ 14''\cdot8$, and the two sides P α and P C are constants, and are equal respectively to $29°\ 54'\ 29''$ and $29°\ 25'\ 47''$. Upon calculating the third side of this triangle, it will be found to be $1°\ 26'\ 12''\cdot6$; that is, it gives for the polar distance of Polaris this value.

Upon examining the *Nautical Almanac* for 1860, we find that Polaris for the 1st January had a declination of $88°\ 33'\ 47''\cdot35$, which gives a polar distance for this star of $1°\ 26'\ 12''\cdot65$, or exactly to one-tenth of a second the value found by our calculation.

Without making any use of the last result, we will calculate the polar distance of Polaris for 1850

from the observations recorded in 1873, and by the means already described.

The angle at C is now increased from 1873 by $41''\cdot03 \times 23 = 15' \, 43''\cdot69$, making the included angle of the triangle α P C=$2° \, 51' \, 5''\cdot09$. Upon working out the third side, we obtain $1° \, 29' \, 24''\cdot4$ for the polar distance of Polaris for January 1st, 1850; and we find in the *Nautical Almanac* for 1850 that the observed polar distance of Polaris on January 1st, 1850, was $1° \, 29' \, 24''\cdot72$; so that we agree by our calculations to within three-tenths of a second of the recorded observation.

We will now venture on a calculation referring to a date no less than fifty-four years in the past, and calculate the Pole-star's distance from the pole at the 1st January 1819. The angle at C is now increased by $41''\cdot03 \times 54$ years, which gives $36' \, 55''\cdot6$; consequently the angle α C P=$3° \, 12' \, 18''$.

Upon calculating the third side of the triangle, we obtain for the polar distance of the Pole-star on January 1st, 1819, $1° \, 39' \, 24''$; and this distance we find recorded in the *Nautical Almanac* for 1819 as having been observed as $1° \, 39' \, 25''$.

We believe we are not claiming too much when we state that to calculate by any sound geometrical process the polar distance of a star to within $1''$ for a date fifty-four years back by any process now known is as much beyond the powers of the present accepted theories in astronomy, as it was beyond the powers of the astronomy of two thousand years ago to calculate

to within a second when an eclipse should occur fifty-four years in advance. Yet here is the fact; it can be done, and is done, and by those very same principles which enable us to calculate the value of the obliquity of the ecliptic to within 1″ for 230 years, and to show an ample cause for the glacial epoch of geology.

The last polar distance of the Pole-star to which we will refer is that given in Bradley's Catalogue for 1755. Upon working out the third side of our triangle for 1755 with the two sides as before and the included angle, we obtain 2° 0′ 20″ for the polar distance of Polaris, whilst Bradley records 2° 0′ 19″. Thus counting back from 1873, we can calculate to within 1″ the polar distance of the Pole-star for no less than 118 years, and we have therefore positive proof that the course traced by the earth's axis is, relative to the pole of the ecliptic and the Pole-star, such a curve as that we have defined it to be. The reader unacquainted with the present powers of astronomical science may not be aware that such a calculation as the above, depending solely on geometry, is at present impossible by any known means, and the reason for its being so is that the true course of the pole is even now unknown to astronomers.

Important as are these results, showing so exact an agreement between calculations based on the conclusion that the true centre of the arc traced by the earth's axis is 6° from the pole of the ecliptic, still we must not omit to notice that there are several im-

portant points yet to be investigated. We know from the calculations already made that an arc P P′ traced by the earth's axis on the sphere of the heavens

varies its distance from a point E in such a manner as to define the arc P P′ as part of a circle having for its centre the point C.

Upon referring this curve to the most important star in the heavens, viz. the Pole-star (represented by α in the accompanying diagram), we again find that the arc P P′ is defined as an arc of a circle having for its centre the point C.

From the fact of knowing what the curve P P′ really is, we are enabled to calculate by a purely geometrical process the distance at any date of the pole of the heavens from the Pole-star; a calculation hitherto unattainable by any astronomer. We are also enabled to calculate the distance at any date of the pole of the heavens from E, the pole of the ecliptic, and thus to ascertain the value of the obliquity of the ecliptic at any date by a sound geometrical calculation; a process hitherto beyond the powers of astronomy.

Whilst it is impossible to ignore the value of these

facts, and to avoid the results which they demonstrate, the following questions have yet to be answered:

First, it is demonstrated beyond question that the arc traced by the earth's axis is part of a circle having for its centre the point localised as 6° from the pole of the ecliptic, and marked C in last diagram; that is, it is part of such a circle when referred to E, the pole of the ecliptic, and α, the Pole-star. There is not, however, as yet any evidence or proof whether the pole of the ecliptic, the pole of the heavens, and the centre of polar motion have, or have not, any uniform motion of their own in any direction; so that whilst they remain, as regards each other, in exactly the same relative position, yet they may or may not alter their position relative to external objects; just as localities on earth, by the earth's rotation, whilst maintaining the same distance from each other, vary their position relative to celestial objects.

Although we have found that the calculated distances of the Pole-star from the pole are such as to give us proof of the arc traced by the earth's axis being part of a circle with a centre C (last diagram), yet if the centre C and the pole P were carried round the Pole-star as a centre, or round a point near it as a centre, whilst the pole was also tracing its arc round C, the conditions relative to the distances C P and α P (last diagram) would be the same as if C were at rest.

This double motion being by some persons con-

sidered difficult to understand, we may make it intelligible if we describe it as the same in effect as the following:

Suppose a railway train to be running round a parallel of latitude on the earth's surface; this train is not varying its distance from the north pole of the earth. Let us now assume the earth to have no rotation, and to be fixed in space as regards the sun; then the railway train would vary its distance from the sun, though only by a small amount. Next let us suppose that, whilst the train is moving in the manner above described, the earth travels round the sun as a centre, but does not rotate; then the railway train would vary in distance from the sun exactly in the same manner as if the earth were at rest.

Thus it is possible that the pole of the heavens, the pole of the ecliptic, and the centre of polar motion may all have a uniform motion in a certain direction without causing any difference in the angular distances from the Pole-star of any one of these three points. *But if such a movement occur, we can at once tell in which direction it can only occur*, in order not to violate the conditions known to exist from our previous calculations.

CHAPTER VI.

CHANGES IN STELLAR LONGITUDES.

IN order that the conditions already demonstrated by calculation should exist, it must follow that if the pole of the heavens, the pole of the ecliptic, and the centre of polar motion have a uniform motion in any direction, it cannot be one causing the pole of the heavens to move directly towards or away from the Pole-star, but it must be a movement causing the pole to move in a lateral direction as regards the Pole-star, even though it may not move round the Pole-star as an exact centre. The movement therefore, if it occur, must be either somewhere in the direction of the arrows shown in the annexed figure or in almost the directly opposite direction.

In order to test whether the pole of the ecliptic has any such motion, we will adopt a method of investigation which does not appear to have been

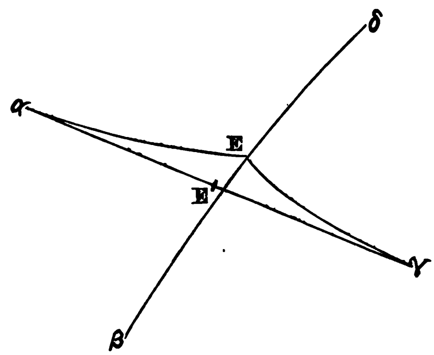
adopted hitherto by any inquirers; but as it is one which is easily understood and easily put in practice, we believe it will be found of considerable use in solving this problem.

It has hitherto been considered that the best means of discovering whether the plane of the ecliptic varied its position relative to the fixed stars was by noting the recorded changes in latitude of those stars which were situated between the equator and ecliptic. There are reasons, however, which prevent this method from being entirely satisfactory; one of these is, that stars on the same meridian of longitude at any one time will not necessarily change their latitude uniformly for any movement of the plane of the ecliptic; so that to check the various supposed changes in one direction by means of the recorded changes in another direction does not give us satisfactory results. We therefore select another method by which to ascertain whether the plane of the ecliptic, and hence the pole of the ecliptic, vary their position in the heavens, and this is by comparing the relative longitudes of certain stars at various dates; for by this method we can discover with tolerable certainty not only the direction, but the amount of change in the position of the pole of the ecliptic.

The process of examination which we will now adopt may be understood by aid of the following diagram:

Let E be the position of the pole of the ecliptic at any date, and α, β, γ, δ, four stars, the longitude of

each of which is known. By taking the differences of longitude of any two of these stars, we obtain the angle at E of the two meridians of longitude passing through these stars; for example, the angle α E β will give us the difference in longitude between the stars α and β; and so on. If, then, we have catalogues of stars showing their longitudes at different dates, we can, by comparing these, discover whether the pole E has changed its position in the heavens and relative to the fixed stars; for if the pole E moved to E′, the

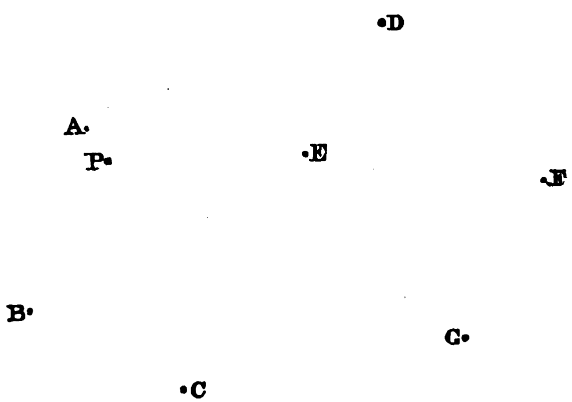


angle α E′ β would be greater than α E β, the angle δ E′ γ would be less than δ E γ; and so on. By the variation in the difference of longitudes at different dates, we have, then, a very simple and practical method by which to discover whether and where the pole, and hence the plane, of the ecliptic moves.

The catalogues of stars in vol. xiii. *Memoirs Royal Astronomical Society* offer us ample means of comparison, as we have therein the latitudes and longitudes of stars given from 140 A.D. to 1660 A.D.

Taking for comparison remarkable and well-known stars, such as Polaris, α Ursæ Majoris, α and β Ursæ Minoris, α Lyræ, Cygnus, &c., we can test, by their respective changes of longitude, the change (if any) in the position of the pole of the ecliptic.

The following diagram will represent approximately the position of those stars which we will make use of in this investigation.



Taking P as the position of the pole of the heavens, E as the pole of the ecliptic, A represents the position of the Pole-star; B, α Ursæ Majoris; C, γ Ursæ Majoris; D, α Cygni; F, α Lyræ; G, Arcturus.

It will now be seen how the relative longitude of stars will vary by any movement of E, the pole of the ecliptic, and independent of any movement of P, the pole of the heavens. The angle A E B, for example, represents the difference in longitude between the stars A and B. If the pole of the ecliptic E moved

anywhere in the direction of A or B, the angle A E B would be increased thereby; whereas if the pole move in the opposite direction, the angle A E B would be decreased thereby. In the same manner, if the pole E move in the direction of G, the angle C E F would be increased thereby. From the catalogues of Ptolemy, Tycho Brahè, and Hevelius we extract the following longitudes of stars with which to make a comparison:

STAR.	LONGITUDE BY PTOLEMY.		LONGITUDE BY TYCHO.		LONGITUDE BY HEVELIUS.		
Polaris	60°	10′	83°	2½′	83°	51′	22″
α Ursæ Majoris	107	40	129	34	130	23	58
γ Ursæ Majoris	123	0	144	45	145	38	2
Arcturus	177	0	198	39	199	29	6
α Lyræ	257	20	279	43	280	33	17
α Cygni	309	10	329	53½	330	42	50
α Aurigæ	55	0	76	16	77	6	51
α Cephei	346	40	7	13	8	9	30
γ Draconis	239	40	262	24	263	9	10

We have here a list of some important stars, and can from it compare the changes, if any, which the evidence indicates to have occurred between 140 A.D. and 1660.

On comparing the difference of longitude between Polaris and α Ursæ Majoris, in Ptolemy's time, we obtain 47° 30′ for the angle at the pole of the ecliptic. The difference by Tycho's observation becomes 46° 31½′, and by Hevelius, 46° 32′ 36″. Consequently, the angle has decreased in 1520 years about 58′; a result which indicates a movement of E *somewhere*

away from that part of the heavens contained by an arc in which these two stars are situated.

Our next comparison between α and γ Ursæ Majoris gives for the angle at E, in Ptolemy's time, 15° 20′, for Tycho's time, 15° 11′; and for that of Hevelius, 15° 14′ 4″; giving a slight decrease in this angle. On comparing γ Ursæ Majoris and Arcturus, we obtain, for Ptolemy's time, a difference of 54°; for Tycho's, 53° 54′; and for Hevelius, 53° 51′ 4″.

Taking our next comparison on stars in the nearly opposite part of the heavens, we find that the angle at E, the pole of the ecliptic, formed by the meridians of longitude passing through Polaris and α Aurigæ, was, in Ptolemy's time, 5° 10′; in Tycho's, 6° 46′ 30″; in Hevelius, 6° 44′ 31″.

The question now presents itself as to what movement of the pole of the ecliptic could occur which would cause the angle at that pole between the stars named to vary in the manner above stated. In order that the angle between Polaris and α Ursæ Majoris should decrease, the pole must move somewhere away from that part of the heavens between 60° and 107° of longitude. To cause an increase in the angle between Polaris and α Aurigæ, a movement of the pole in the opposite direction is required. If such a movement of the pole of the ecliptic has occurred as is indicated by Polaris and α Ursæ Majoris, we ought to find that the angle at the pole of the ecliptic, between stars on the opposite side of this pole to that on which Polaris is situated, will have increased.

On comparing the angle between α Lyræ and α Cygni, we obtain for it at 140 A.D. 51° 50′; for Tycho's time, 50° 10½′; for Hevelius, 50° 9′ 33″; showing a decrease just as would occur if the pole of the ecliptic be moving towards the opposite part of the heavens to that in which these stars are situated. On comparing the angle between γ Draconis and α Cephei, we obtain 107° 0′, 104° 49′, and 105° 0′ 20″; showing a most marked decrease in this angle, which would occur by the movement of the pole away from these stars.

By taking small stars nearer the pole of the ecliptic, we ought to obtain greater differences in the angles than when stars distant from the pole are selected. The stars γ Cephei and β Cephei serve well as test stars, and these we find given by Ptolemy, 33° 0′ and 7° 20′ longitude; by Tycho, 54° 23′ and 30° 13′; by Hevelius, 55° 24′ 49″ and 30° 58′ 30″. The differences, or angle at E, will stand thus: by Ptolemy, 25° 40′; by Tycho, 24° 10′; by Hevelius, 24° 25′ 19″. Such a change as the above would indicate that the pole of the ecliptic had moved away from that part of the heavens in which the stars above named are located.

The stars δ and β Ursæ Minoris are given the following longitudes:

By PTOLEMY, 62° 30′ and 107° 20′. Difference, 44° 50′.
By TYCHO BRAHE, 85° 36′ and 127° 16½′. Difference, 41° 40½′.
By HEVELIUS, 86° 29′ 40″ and 128° 21′ 8″. Difference, 41° 52′ 28″.

This decrease in the angle at E would indicate that

the pole of the ecliptic had moved away from that part of the heavens contained between the two meridians passing through δ and β Ursæ Minoris.

From comparisons of the longitude of γ Draconis and α Cygni we obtain 69° 30′ for 140 A.D., 67° 29½′ for Tycho's time, and 67° 33′ 40″ for Hevelius.

If these comparisons and results prove anything, they prove that the angle at the pole of the ecliptic formed by two meridians passing through certain stars decreases in almost every instance, no matter where the star is situated; a result which cannot be explained by any movement of the pole of the ecliptic in any particular direction in the heavens. If the pole of the ecliptic has during the past 1500 years changed its position in the heavens, it must have moved from some other position to that it now occupies. If it has moved from some other position, it must have caused the angle at that pole formed by the two meridians passing through two stars to increase or decrease according as the pole has moved towards or away from such stars. If the angle thus formed by two stars in one direction *increase*, then the angle formed by two stars in the opposite direction *must decrease.* Yet from an investigation of facts we find no such increase and decrease clearly demonstrated, as would be demonstrated if the pole of the ecliptic had changed its position in the direction, to account for the variation in the obliquity. Now, the importance of this fact must not be lost sight of. Modern theorists assert that the pole of the ecliptic is the centre of

the circle traced by the earth's axis, and that the decrease of the obliquity of the ecliptic is satisfactorily explained, in consequence of the pole of the ecliptic having a slow motion towards the pole of the heavens. We state that the first assumption is contradicted by the second; that it is impossible that the centre of a circle can change its distance from the circumference; and we also bring forward the recorded observations of 1520 years to show that there is no evidence to indicate that the pole of the ecliptic has a motion towards that particular direction.

It follows, then, that the present accepted theory of the cause of the decrease in the obliquity of the ecliptic—viz. that the pole of the ecliptic moves towards that part of the heavens in which the pole of the heavens is situated—is not corroborated by facts during 1520 years; and therefore the phenomenon of the decrease in the obliquity is, from these evidences, in reality without any sound explanation, except that of the movement of the pole of the heavens around a circle the centre of which is 6° from the pole of the ecliptic.

As, however, this question is one of great importance, we must not be content without a full and searching investigation into recorded facts before we bring to bear those geometrical laws which will materially affect the problem.

CHAPTER VII.

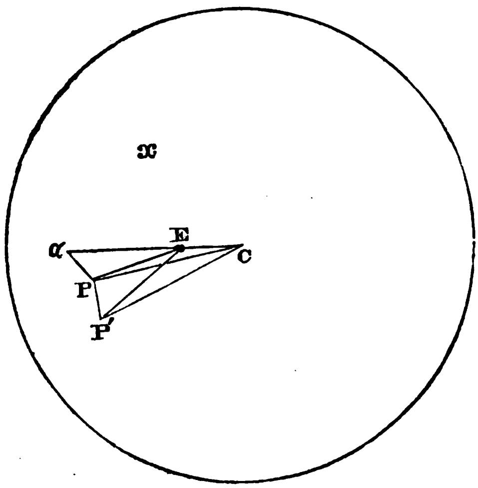
COURSE OF THE POLE OF THE HEAVENS AS SHOWN BY THE POLE-STAR.

In order to find the polar distance of Polaris for any date, we made use of three points, viz. the star itself, the centre of polar motion, and the pole of the heavens. The angle we made use of was the angle formed at the centre of polar motion by two meridians, one of which passes through Polaris, the other through the pole of the heavens. We have made no use of the right ascension of the star, except to determine for 1873 the angle formed at the pole of the heavens by the two meridians passing respectively through the star and the centre of polar motion, and we have not therefore in this calculation made any reference to the pole of the ecliptic; it is of no consequence as affecting these results of polar distances whether the pole of the ecliptic does or does not vary its position in the heavens.

From the calculations already given we have a very simple means of testing, not only whether the pole of the ecliptic is a fixed point, but also whether the pole of the heavens and the centre of polar motion are also fixed. In order that the reader may under-

stand this investigation, we must again refer to the data on which we have already been able to calculate the obliquity of the ecliptic for any date and the polar distance of the Pole-star.

In the following diagram we take, as before, C the centre of polar motion, E the pole of the ecliptic and the Pole-star, P the position of the pole of the heavens at the date 1873, P′ the position of the pole of the heavens at any other date in the past; P′ C=P C.



We have the following values for the date 1873: C E=6°, C P=29° 25′ 47″, P α=1° 22′ 4″·3; the angle α P E=108° 4′ 26″; the angle E P C=1° 15′ 44″·6; and P E=23° 27′ 20″·8.

Upon taking P′ as the position of the pole for the date 1860 A.D., we find by calculation P′ α= 1° 26′ 12″·6. We can also find by the same date the value of the side P′ E, the value of the angles E P′ C,

α P′ C, and consequently α P′ E; for we have C E=6°, C P′=29° 25′ 47″. The angle E C P′=41″·03 × {2295·5−1860}; consequently we can find E P′ and the angle E P′ C, for we have two sides and the included angle of the spherical triangle E C P.

Upon calculating the value of these items, we obtain 23° 27′ 27″ for the arc P′ E, which agrees to a second with the obliquity observed and recorded for 1860, though we make use of no recorded observation later than the date 2295·5 A.D., or 435·5 years in advance; and for the angle E P′ C we obtain 1° 18′ 7″. Now the angle α P′ C for 1860, when calculated by means of the two sides P′ C and C α, and the included angle α C P′, gives 108° 15′ 1″; consequently α P E=α P C−E P′ C=106° 56′ 54″; from which we subtract 90°, leaving 16° 56′ 54″ for the AR of α at 1860.

On reference to the *Nautical Almanac*, 1860, we find the AR of Polaris given for the 1st of January, 1 h. 8 m. 2·61 s., which, converted into arc, gives 17° 0′ 39″·15; consequently the angle α P′ E for 1860 would be, by observation, 107° 0′ 39″·15, or 3′ 35″ greater than it would be if the points E and C were fixed.

In order that the angle α P′ E should be larger, it would be necessary for P′ to have moved with E and C in the direction of C P′ produced; or if C remain fixed, then the same effect would be produced if E had a small movement in the direction of x. A movement of E in the direction of P′ would not

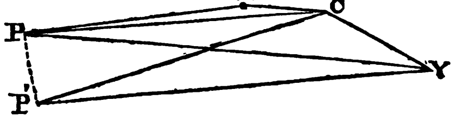
cause the angle α P′ E to vary; *a fact of great importance.*

Hence, to fulfil the known conditions as regards the pole of the heavens, the polar distance of Polaris, the obliquity of the ecliptic, and the angle formed at the pole of the heavens by two meridians passing through Polaris and E, the pole of the ecliptic, the centre of polar motion, and the pole of the heavens might have an uniform rotation or movement round any point situated on any meridian near one or six hours' AR, or anywhere between these meridians, or they might have a slow rotation round a point with eighteen hours' AR.

In order to bring as many checks as possible on this problem, we will select another important star, viz. γ Draconis, and work in the same manner, as regards this star's distance, from the pole of the heavens and the centre of polar motion.

From the *Nautical Almanac*, 1873, we obtain the following data for γ Draconis: declination, 51° 30′ 16″·1; right ascension, 17 h. 53 m. 39·431 s.

The following figure will represent the position on the sphere of the heavens of γ Draconis relative to the pole of the heavens, the centre of polar motion, and the pole of the ecliptic.



C represents the centre of polar motion, P the pole

of the heavens, E the pole of the ecliptic at the date 1873 A.D., γ the position of γ Draconis. We have C E$=6°$, C P$=29° 25' 47''$, P $\gamma=38° 29' 44''$, the polar distance of γ Draconis, 1873. The angle E P $\gamma=270°-$ the right ascension of γ Draconis$=6$ m. 20·57 s.$=$in arc to $1° 35' 8''\cdot5$. The angle E P C for 1873$=$ $1° 15' 44''\cdot6$; consequently C P $\gamma=$E P $\gamma-$E P C$=$ $0° 19' 23''\cdot9$. We have, then, P C and P γ, and the included angle C P γ, to find C γ, which, by calculation, is found to be $9° 4' 2''$; whilst the angle P C γ is $178° 43' 25''$.

To find the polar distance of γ Draconis for 1860 we then have P$'$, the position of the pole of the heavens for 1863, P$'$ C$=29° 25' 47''$, C $\gamma=9° 4' 2''$, and the angle P$'$ C γ.

The angle P$'$ C $\gamma=$P C $\gamma-$P$'$ C P$'$, and P C P$'=13$ years $\times$ $41''\cdot03=8' 53''\cdot39$. We have, then, two sides, P$'$ C and C γ, and the included angle P$'$ C γ, to find P γ.

Upon calculating the side P γ, we obtain $38° 29' 38''$ for its value in 1860, whilst we find recorded in the *Nautical Almanac* $51° 30' 24''\cdot2$ as the declination; therefore $38° 29' 35''\cdot8$ as the value of the arc P γ by observation in 1860. Consequently the difference between observation and calculation is only $2''\cdot2$; a result which, considering the smallness of the angles of the triangle, is very remarkable.

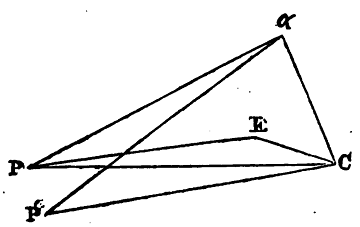
Upon working out the same problem for 1755, we obtain by calculation $38° 29' 10''$ for the polar distance of γ Draconis, whilst by observation it was

38° 29′ 19″·4, or 9″·4 more by observation than by calculation.

From these results it is quite possible that the pole of the heavens, the pole of the ecliptic, and the centre of polar motion have a uniform slow motion laterally as regards Polaris and γ Draconis, and it is quite possible that whilst such a movement does not affect the polar distance of Polaris, it does affect slightly the polar distance of γ Draconis, because the two stars are not separated by exactly 180° of right ascension.

If, however, the pole of the heavens, the pole of the ecliptic, and centre of polar motion have a uniform motion laterally as regards Polaris and γ Draconis, then this movement must be direct as regards those stars at right angles to the arc joining Polaris and γ Draconis. For example, for such a star as α Cephei or γ Ursæ Majoris we ought to find much larger differences than we do with γ Draconis.

Adopting the same method of calculation for the two stars α Cephei and γ Ursæ Majoris we have for the former the following case:



P position of pole at 1873, E pole of ecliptic, C centre of polar motion, α the star α Cephei.

The angle α P E=right ascension of $\alpha-270°$, P α =polar distance of α 1873, E P C$=1° 15' 44''\cdot6$, C E$=6°$.

From the above we have in the spherical triangle α P C, P C$=29° 25' 47''$, P $\alpha=27° 57' 8''$, α P C$=50° 8' 57''$, to find the constants C α, and the angle α C P.

From these data we obtain C $\alpha=23° 31' 6''$, and angle α C P $64° 23' 52''$ for 1873.

In order to find P′ α for 1755, we have C α and C P′ constants, and the angle α C P′$=\alpha$ C P+P C P′. The angle P C P′$=41''\cdot03\times\{1873-1755\}=1° 20' 41''\cdot5$. We have then, as before, two sides and the included angle, to find the third side P′ α.

The side P′ α, by calculation, is $28° 27' 22''$; whilst, from Bradley's Catalogue, 1755, we find it was by observation $28° 26' 42''\cdot6$. That is, the pole was $39''\cdot4$ nearer the star in 1755 than it would have been had there been no additional movement in the pole P other than its circular course round C at the rate of 20·158 per annum. To account for this variation of $39''\cdot4$ in 118 years would require that from 1755 to 1873 there was a movement of the pole of $39''\cdot4$ *away from* the star α Cephei, or at a rate of $0''\cdot33$ per annum.

As the star α Cephei has a right ascension of about 20 hours 37 minutes, it would follow that the movement of the pole ought to be somewhere towards 8 hours' right ascension; and such a movement would be quite in harmony with the fact that both Polaris

and γ Draconis showed little or no change of movement in the pole other than its circular course round the centre C.

Upon pursuing the same investigation with regard to δ Ursæ Minoris, we find a similar result to that indicated by γ Cephei. This star has about 18 hours 13 minutes' right ascension, and for its constants for 1873 we have 29° 25′ 47″, and 26° 2′ 56″ for the two sides of the triangle, whilst the included angle at C is 37′ 4″·6. Correcting the angle at C for 1755, we obtain, for the polar distance of this star for 1755, 3° 30′ 6″, whilst, by observation, it was found 3° 29′ 8″·5. This would indicate a movement of the pole away from this star of 57″·5 in 118 years. We now take a star on the opposite side of the pole, having about twelve hours' right ascension, and take γ Ursæ Majoris as such star. Adopting similar calculations for this star to those adopted in the former cases, we find that the polar distance in 1755 would have been 34° 55′ 34″, but was recorded as 34° 56′ 36″. The pole was therefore 1′ 2″ *further* from the star in 1755 than it would have been if there were no additional motion in it other than its circular course round C.

From the above facts there appears strong evidence to indicate that the pole of the heavens has an independent general motion towards that part of the heavens situated somewhere between 8 and 12 hours' right ascension, such a motion as would be produced by a slow movement of the plane of the ecliptic, carrying with it the earth and earth's axis round an

axis directed towards about 4 hours' right ascension, or such a movement as would be produced by a slow revolution of the pole of the ecliptic (carrying with it the pole of the heavens) round the centre of polar motion, but in the opposite direction to that followed by the pole of the heavens; so that, taking C as the centre of polar motion, E as the pole of the ecliptic, and P as the pole of the heavens, it appears from the above

evidence that whilst P moves round C in the direction from P to P' at the rate of 41″·03 per annum when referred to a great circle, E appears to move round C in the direction of E E'.

If such a movement occur, it would follow that stars having about 120° to 210° longitude would have more latitude in modern times than they had in the days of Ptolemy; because if E, the pole of the ecliptic, has approached such stars, their north latitude must have increased. It is a fact, that although there are several discrepancies, due probably to errors of observation or imperfect instruments, yet the seven principal stars of the Great Bear have all much greater latitude in modern times than Ptolemy assigned them in 140 A.D., the increase being as follows:

	GREATER LATITUDE IN 1660 THAN IN 140 A.D.
α Ursæ Majoris	40′
β "	37′

	GREATER LATITUDE IN 1660 THAN IN 140 A.D.
γ Ursæ Majoris	38′
δ "	38′
ε "	49′
ζ "	43′
η "	25′

Whilst the stars of Cassiopeæ in the opposite part of the heavens have nearly all less latitude in 1660 than in 140 A.D.

From the above evidence it appears that, whilst the pole of the heavens describes a circle round the point C 6° from the pole of the ecliptic and 29° 25′ 47″ from the pole of the heavens, and thus accounts for the precession of the equinoxes and the variation in the obliquity, the pole of the ecliptic has a slow motion somewhere in the direction of that part of the heavens included between 120° and 210° of longitude, in which motion the pole of the heavens partakes.

CHAPTER VIII.

ON CERTAIN GEOMETRICAL LAWS AFFECTING THE CHANGE IN STARS' RIGHT ASCENSION AND DECLINATION.

FROM the few remarks contained in the preceding chapters it will be seen that our earth is, to all intents and purposes, a large instrument, which the astronomer makes use of in order to determine the position of various celestial bodies at various times. This instrument rotates during every twenty-four hours round an axis which does not perceptibly change its direction during so short an interval of time as twenty-four hours; it also moves round the sun during a year; and these two movements recur without any apparent change. The earth rotating on its axis with uniformity, and revolving round the sun with apparent uniformity.

The movement of the earth, which has been termed the 'conical motion of the axis,' is one so slowly performed that, even during the whole history of astronomy, not one-tenth of this movement has been completed, and it becomes therefore a problem for investigation; for it is not a completed movement, as is the rotation of the earth, or its revolution round the sun.

The axis of a sphere is, geometrically speaking, a

line, but it is a line fixed in the sphere; and although we may be able to delineate the course traced or marked out by this line on the sphere of the heavens, yet a considerable amount of farther investigation is necessary in order to find in what manner the sphere itself moves in consequence of the movement of the axis. It will be demonstrated at a future page that there are several very different movements of a sphere, each of which would produce apparently exactly the same movement of the axis of the sphere. Consequently, although we may have traced out with every detail the movement of the axis of a sphere, yet we may remain totally unacquainted with the movement of the sphere which accompanies the change in direction of the axis.

In the following demonstrations we first treat of the results due to a change in direction of the earth's axis, and the effects produced on the assigned position of various stars. After having shown some of the geometrical laws connected with this movement, we will next explain how various changes of the sphere (that is, of the earth) may occur whilst the axis itself moves in identically the same manner.

From the observations of many centuries it has been found that the earth's axis changes its direction annually to the amount of about $20''\cdot 15$.

That point in the heavens towards which the earth's axis is directed is called the pole of the heavens; consequently the pole of the heavens alters its position annually to the amount of about $20''\cdot 15$.

Observation has made us acquainted with the fact, that the pole of the heavens appears to decrease its angular distance about $20''\cdot15$ from those stars which are situated near the meridian of 0 hour's right ascension, and to increase its angular distance from those stars which are situated near the meridian of 12 hours' right ascension. From these observations, carried on during several hundred years, it is known that the *approximate* direction of the movement of the pole is towards that part of the ecliptic which, for the time being, is on the meridian of 0 hour's right ascension.

To obtain an approximation to any result is simple; to arrive at great accuracy is the work of labour, time, and thought. To know nearly in which direction the pole of the heavens was moving was easy; to discover exactly in which direction it moves was a work of time hitherto unaccomplished.

If the pole of the heavens always moved exactly towards that point on the ecliptic which for the time being was on the meridian of 0 hour's right ascension, it would trace an exact circle round the pole of the ecliptic as a centre, and consequently would always maintain the same distance from that pole. Observation, however, has shown that, during the past eighteen hundred years at least, the pole of the heavens has gradually decreased its distance from the pole of the ecliptic; whilst it has moved onwards $20''\cdot158$ per annum.

Referring to the annexed diagram, the course

traced by the pole of the heavens relatively to the pole E of the ecliptic is shown by the curve P O R S,

which curve, as it moves from P to S, decreases its distance from E. Now, as the rate of the decrease of the various portions of the curve P S from E are known, it follows that the true character of the curve P O R S can be discovered when this curve is referred to the point E.

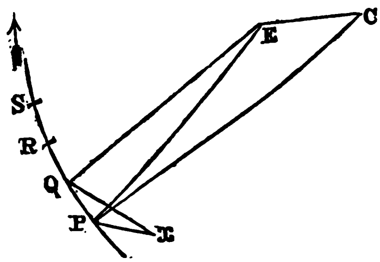
From an investigation of this curve it is found that all the conditions are fulfilled relative to the distances of various portions of the arc P S from E, when we define this arc as a portion of a circle the centre of which is 6° from E the pole of the ecliptic, and so situated that at the date 2295·5 A.D. this centre, the pole of the ecliptic, and the pole of the heavens will be on the same great circle of the sphere.

To trace out the true nature of this curve is a matter of the greatest importance, although it is only one step towards a solution of those problems affecting the changes in stars' positions; for even when we know the exact course traced by the earth's axis on the sphere of the heavens, yet we have still to ascertain the movement of the sphere which accompanies this course of the pole. We can, however, refer to many important problems when we know the course of the pole of the heavens as regards the pole of the ecliptic, even though we have not as yet defined the

remainder of the mechanism connected with this motion.

In our last work, viz. *The Cause, Date, and Duration of the last Glacial Epoch*, some evidence was given showing on what grounds the circle traced by the pole of the heavens was such as we have described it to be. As the proof therein given was a geometrical one—and such a proof is undeniable—we will here only briefly refer to the fact of this movement, and then describe the effects it produces on the changes in the right ascension of stars.

Referring to the accompanying diagram, the circle



of which P Q R S is an arc is the circle traced by the pole of the heavens. The centre of this circle is C, 6° distant from E, the pole of the ecliptic; S is the position which the pole will occupy at the date 2295·5 A.D.

The movement of the pole of the heavens is at right angles to the arc joining P and C when the pole is at P; it is at right angles to the arc joining Q and C when the pole is at Q, and so on. The length of the radius P C is 29° 25′ 47″, and as E C is 6°, S E will be 23° 25′ 47″.

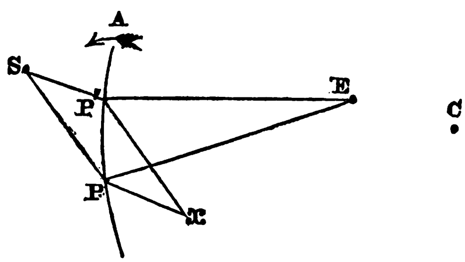
When describing the method of framing a catalogue of stars, and assigning to each a certain right ascension, we pointed out that the right ascension of a star was measured by the angle at the pole between two meridians, one passing through the star, the other through the first point of Aries. The angle formed at the pole between two meridians, one passing through the first point of Aries, the other through the pole of the ecliptic, will be 90°. Therefore, the right ascension of the polc of the ecliptic may always be taken as 270°—that is, six hours short of the zero of right ascensions—and it will be therefore always eighteen hours, or 270°.

As the changes produced in the right ascension of stars, and due to the movement of the pole of the heavens, do not appear to be generally understood, we will describe these changes as regards their geometrical results, and point out certain laws connected with these changes, which laws seem hitherto to have been overlooked by astronomers.

The effect of the movement of the pole of the heavens along the circumference of a circle the centre of which is 6° from the pole of the ecliptic is to cause a large majority of stars to increase their right ascension annually; a few stars situated in a particular part of the heavens to decrease their right ascension annually; whilst a very small number of stars would neither increase nor decrease their right ascension. We will first show how it is that some stars increase, whilst others decrease, their right ascensions in

consequence of the movement of the pole of the heavens.

In the annexed diagram P represents the position of the pole of the heavens at any date, E the position of the pole of the ecliptic, S and X two stars.



The meridian of which PE is a part will be a meridian of 270° right ascension; and as the earth's rotation is from right to left (as shown by the arrow at A), the star X will pass the meridian before E passes. The right ascension of X will be 270°—the angle E P X. The second star, S, will pass the meridian after E, the interval after being measured by the angle E P S. Thus the right ascension of S will be 270°+ the angle E P S; and if E P S were 120°, then the right ascension of S would be 30°, viz. {270° + 120°} — 360°.

The pole which was at P we will now suppose to have moved to P′ in its circular course in the heavens. The arc joining P′ and E would now be part of the meridian of 270° right ascension. The star X would now pass the meridian before E by an interval of time measured by the angle E P′X; and as it is evident that the angle E P′X is greater than E P X, it follows that 270°—E P′X is less than 270°—E P X; that

is, the right ascension of the star at X has decreased in consequence of the movement of the pole from P to P′.

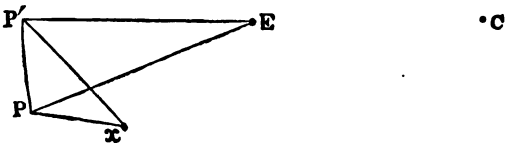
The star at S will pass the meridian when the pole is at P′ after E, by an interval of time measured by the angle E P′ S; and as E P′ S is greater than S P E, it follows that 270° + E P′ S is greater than 270° + E P S. Consequently the star at S has increased its right ascension by the movement of the pole from P to P′. Here, then, there are two stars, viz. X and S, one of which increases, the other decreases, its right ascension by the movement of the pole from P to P′.

By following out the principles here demonstrated, and placing stars at various points on the diagram, the reader will be able to discover for himself whether a star's right ascension increases or decreases; and it will be found that stars between the pole of the heavens and the pole of the ecliptic will all decrease their right ascensions, whilst those in almost all other parts of the heavens will increase their right ascensions. The detail values of these changes, and other items connected therewith, will be shortly given.

We now call attention to the important fact that the initial meridian from which right ascensions are counted is that meridian which passes through the first point of Aries; also that the meridian of 270° (*i.e.* eighteen hours') right ascension always passes through the pole of the ecliptic. Hence, as the pole of the heavens (P in last diagram) moves round the circumference of the circle to P′, it *drags* with it the

meridian of 270° right ascension, and thus causes various stars to change their right ascensions. That the pole of the ecliptic E is not the centre of the circle traced by the pole P of the heavens is not directly discoverable, in consequence of this change in right ascension of various stars. Even if the radius of the circle described by P, the pole, happened to be three or even four times the length of the arc P E, and the centre were in the direction of the arc P E produced, yet the change in the position of the pole from P to P′ would still cause the 270° meridian of right ascensions to vary from P E to P′ E. That other variations must occur will be evident to the merest tyro in geometry; and of these we will treat at a future page.

It being essential that the reader should thoroughly comprehend this special portion of the problem, we will take an extreme case as an example, and we will suppose P P′ the movement of the pole of the heavens during a given period, the arc P P′ being part of a circle, the centre of which is C; E represents the pole of the ecliptic, and X a star.



When the pole was at P, the star X would have a right ascension of 270°—E P X; and when the pole was at P′, the star X would have a right ascension of 270°—E P′ X.

From the change in right ascension of X no astronomer would discover that E was not the centre of the arc traced by P P′. And as the variation in the angular distances of P E and P′ E would be scarcely perceptible in consequence of the minuteness of this variation, it follows that the fact of C, and not E, being the centre of the arc P P′ must be discovered, in consequence of some discordances which would occur from the true position of this centre not being at E. It will be, of course, evident that although the meridian P E would be a meridian of 270° when the pole was at P, and the meridian P′ E one of 270° when the pole was at P′, yet exactly the same results could not happen if C were the centre of the arc P P′ that would happen if E were the centre.

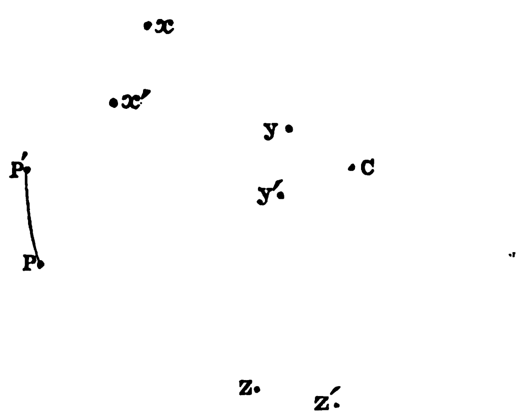
When, therefore, we have discovered that the pole of the heavens changes its position about 20″·15 per annum, and that it moves almost directly towards the first point of Aries, we have only solved a very small portion of the problem connected with the movement of the pole, although the small portion we have discovered gives a cause for the precession of the equinoxes, and for the approximate changes observed in the right ascension and declination of the various stars.

When treating of motion as regards various bodies, we may sometimes render a description more simple if we reverse the actual motion of two bodies, and consider the moving body to be stationary, and the stationary body movable; for example, it is

easier to understand the rising and setting of the sun if we speak of it as the sun rising in the east, passing south of us and setting in the west, instead of speaking of the sun as stationary and of the earth rolling round from west to east, and thus causing the effect referred to.

This method of explanation we have found particularly applicable to the changes which occur in connection with the right ascension of the stars, and due to the movement of the pole of the heavens; for if we speak of the pole of the heavens as stationary, and of the stars as though they all slowly revolved round a point in the heavens, this point being the same as that round which the pole moved, the effects would be almost identical, sufficiently so, at least, to illustrate one portion of this problem.

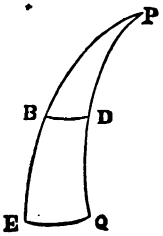
Let us suppose, for example, that in the following diagram P is the pole of the heavens, and describes



a circle round C in a given time; that X, Y, and Z are

three stars. Let us suppose that the pole P moves from P to P′, and thus altered its position relative to X, Y, and Z. If, instead of the pole moving from P to P′, the stars X, Y, and Z themselves revolved round C, their changes of position as regards P and the arc P C would be identically the same as if P moved and the stars remained fixed. This is why the olden astronomers, who mistook the pole of the ecliptic for the true centre of the circle traced by the earth's axis, described the polar movement as the same in effect as if the whole heavens slowly revolved round the pole of the ecliptic as a centre. The effects would be the same if the pole of the ecliptic were the centre of the circle traced by the earth's axis; but as it is not the centre, the effects are very different.

For the sake of illustrating some of the changes in the value of the right ascension of various stars, and due to the movement of the pole of the heavens, we will describe the results as if the stars themselves moved; for by such an arrangement we can more easily point out how the various values of right ascension become altered. Prior to this demonstration we must remind the reader of the method of finding the angle at the pole, formed by two meridians when the value of the arc joining the meridians is known,* and the distance of



* Take P as the pole of a sphere. P D=P B; P Q=P E = 90°. Then the value of the arc B D may be found from the equation,

$$E\,Q = \frac{B\,D}{\sin.\,P\,B.}$$

the arc from the pole is known. We shall be compelled to make use of this problem so frequently in future demonstrations that the reader is advised to acquire a knowledge of it in order to understand the investigations.

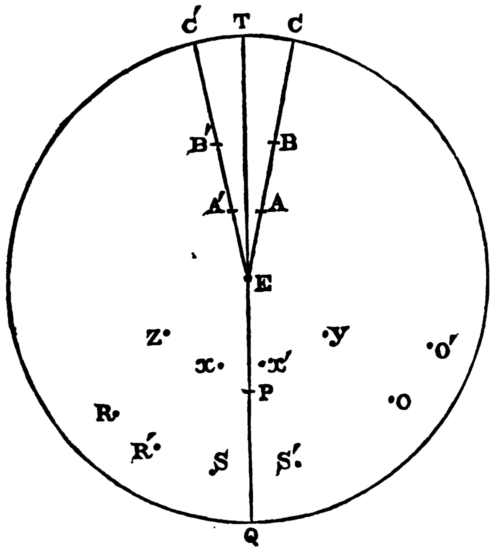
As in the next few pages we merely wish to point out certain laws connected with the actual or relative changes in the right ascension of stars, we will describe these laws on the supposition that the pole of the ecliptic is the true centre of the circle traced by the pole of the heavens; that the rate at which the pole moves is about 1° in 180 years, when measured on a circle about 23° 28′ from the pole of the ecliptic. Neither of these suppositions is, however, quite correct, as the reader is already competent to understand; but it will simplify our demonstrations to assume them to occur for the present.

Referring to the following diagram, we take E as the pole of the ecliptic, C′ C Q as the ecliptic, P the pole of the heavens, A, B, C, O, S, R, stars at various distances from E and P.

When the pole was at P, the right ascension of E would be 270°; the stars A, B, C, would have a right ascension just short of 270°; the meridian of 270° being P E T.

Let us now suppose the pole P to move 1° in its circular course round E; but this movement we will represent by an apparent rotation of the heavens round E.

E P = 23° 28′, and we can then calculate the arcs



passed over by the stars R, O, A, B, &c. The amount of polar movement being 1°, and the arc E P being 23° 28′, it follows that the greatest arc, 90° from E, can be found from the equations

$$E\,Q = \frac{B\,D}{\sin.\ P\,B} = \frac{1^\circ}{\sin.\ 23^\circ\ 28'}.$$ See Diagram, p. 108.

Omitting seconds, this would give for E Q 2° 30′. That is, for a polar movement of 1° there would be a movement of the ecliptic of 2° 30′. This 2° 30′ is the precession which would follow a polar movement of 1°; and it will enable us to find out the extent of the arc which each star would appear to move over if we considered the motion of the pole represented by an apparent rotation of the heavens.

Thus, suppose A a star having a colatitude of say 10° = E A, and let A A′ represent the arc over

which this star appears to be carried by the polar movement of 1°, or rather, by an *apparent* motion of the ecliptic of 2° 30′; making use of the equation,

$$E\,Q = \frac{B\,D}{\sin.\ P\,B}$$

we have B D = E Q sin. P B; substituting, we have

$$\begin{aligned} A\,A' &= 2^\circ\ 30' \times \sin.\ 10^\circ \\ &= 26'\ 2''{\cdot}7 \end{aligned}$$

Now this star is so near the meridian of 18 hours' right ascension, that the polar distance P A would be equal to P A′; and this polar distance would be (P E + E A) 23° 28′ + 10° = 33° 28′ nearly.

Now what is the value of an arc of 26′ 2″·7 33° 28′ from the pole of the heavens when expressed in terms of a diurnal rotation?

One degree being equal to 4^m at the equator, we have, by the same formula as before, B D = E Q sin. P B; and substituting, we have

$$\frac{A\,A'}{\sin.\ P\,B} = E\,Q = 1^\circ\ 30' = 6\text{m}.$$

Hence this star A has increased its right ascension 1° 30′ by the movement of the pole of 1°.

Let us take another star, B. Suppose this star 25° from the pole of the ecliptic, and close to the meridian of 18 hours' right ascension; proceeding as before we obtain for the arc B B′ over which the star moves 2° 30′ + sin. 25° = 1° 3′ 33″. To convert this arc into terms of a diurnal rotation we have, as before,

$$\frac{B\,B'}{\sin.\ B\,P} = \text{Polar angle}$$

$$\frac{1^\circ\ 3'\ 33''}{\sin.\{25^\circ + 23^\circ\ 28'\}} = 1^\circ\ 24'\ 53''$$

Thus the star at B would have increased its right ascension 1° 24′ 53″ in consequence of the polar movement of 1°.

Let us now take a star with about 6 hours' right ascension, and also with a colatitude of exactly 25°, and let S be such a star.

The effect on all stars equidistant from the pole of the ecliptic would be the same as regards the length of the arc over which the star appears to be carried; thus the arc S S′ would be 1° 3′ 33″. Now this star would be distant from P, the pole of the heavens, rather more than 25° − 23° 28′ = about 1° 32′. It would at the point midway between S and S′ be exactly 1° 32′ from the pole; but at S and S′ this value would not be minutely accurate. As, however, the distances P S and P S′ would be equal, we may during this illustration neglect this trifling difference, and consider the star's change of right ascension, in consequence of its movement over an arc of 1° 3′ 33″, at 1° 32′ from the pole.

Making use of the equation as before, we obtain

$$\frac{S\,S'}{\sin. 1^\circ\ 32'} = 39^\circ\ 41'\ 13''$$

It will be seen from this result how great a change is produced in the right ascension of this star near the pole. It is because the diurnal rotation is so slow near the pole that this effect is produced. Thus, for a given polar distance of a star, the change in right ascension annually is greatest for stars between P and Q, that is, stars having 6 hours' right ascension,

and least for stars between E and a point on E T 90° from P.

Let us suppose a star with 18 hours' right ascension and on the equinoctial, and calculate the change in that star's right ascension due to a polar movement of 1°.

Such a star would be only $90° - 23° \ 28' = 66° \ 32'$ from E. Then, as before, we obtain by the equation, for the arc over which the star would appear to move, $2° \ 17' \ 35''$, and as this arc is on the equinoctial, it represents the polar angle, or change in right ascension, of the star.

We have given all these values for a supposed polar movement of 1°. Now, as the polar movement is about $20''\cdot15$ annually, or in round numbers, 1° in 180 years, it follows that, if we divide the total amount of change in right ascension shown for 1° by 180, we shall obtain the value of the change for one year very closely.

Thus for the arc of the equator, or for a star on the equator which has 18 hours' right ascension, we have

$$\frac{2° \ 17' \ 35''}{180} = 45''\cdot81 \text{ nearly}$$

for the arc passed over by that star annually, on the supposition of the pole of the ecliptic being the true centre of polar motion, and the obliquity being 23° 28′.

If we work out this item with greater accuracy, taking 23° 27′ 20″ for the obliquity, and 179·7 years

to make one degree of polar movement, we obtain $46''\cdot12$ for this annual change.

Without at present entering on these minute details, we will merely remind the reader that this value can readily be calculated when we know the exact value of the arc which the pole of the heavens traces, and the exact obliquity of the ecliptic.

We will now show the annual change in right ascension per annum of the stars which we have located on diagram, page 110.

First, let us take the star A, that in 180 years has altered its right ascension $1° 30'$; that is, $90'$ in 180 years, or $30''$ annually. Now $30''$, represented in time, is $2^s\cdot0$. Thus a star which has a right ascension of 18 hours and a north polar distance of $33° 28'$ would have an annual increase in right ascension of $2^s\cdot0$.

Converting this north polar distance into declination, we have for this star's declination $56° 32'$. On looking on the sphere of the heavens, we find no star situated exactly at this spot; but there is a star but little removed from the other position that we have selected for the supposed star B, viz. B.A.C.* 6218 Lyræ. This star has a north polar distance of about $49° 7'$, and a right ascension not far from $18^h 21^m$. The star at B which changed its right ascension $1° 24' 53''$ in 180 years would, at this rate, change its right ascension $28''\cdot3$ per annum, which arc, converted into time, gives $1^s\cdot88$.

* B.A.C. stands for British Association Catalogue.

Now, on reference to B.A.C. or to Loomis's Catalogue of 1500 stars, it will be seen that the annual variation in AR* of B.A.C. 6218 Lyræ is $1^s{\cdot}915$.

Without crowding these pages with these mere elementary matters, we will give only two more examples of this method of calculating changes in stellar right ascensions, and thus show that, given the obliquity and the colatitude and polar distance of any star for one date, then knowing the polar movement in arc per annum, the star's right ascension can be calculated for any other date by spherical trigonometry; for although, for the sake of simplicity, we have given only those stars near 18 and 6 hours' right ascension, still the same principle holds good for every star in the heavens.

Thus, for example, let R be a star's position at any date. Then, owing to the polar movement—which may be transferred, as far as motion is concerned, into a rotation of the sphere round E—this star would appear to be carried to R′, E R being equal to E R′. Then R P R′ would be the hour angle representing this star's change of right ascension, owing to its apparent movement from R to R′.

Let O be another star which would appear to be carried to O′, then O P O′ would represent this star's change in right ascension due to a given polar movement. It will be evident that near 0 and 12 hours' AR there will be an almost similar change in right ascension between two stars whose distance from E

* AR means right ascension.

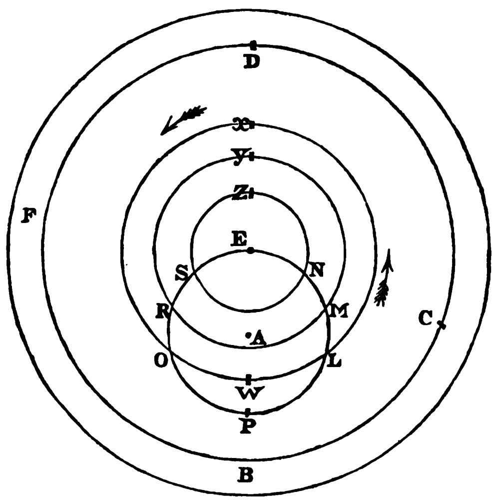
is similar; but this is not the case with stars at 18 hours' and 6 hours' AR, as has been already demonstrated with regard to the stars supposed to be at S and B, diagram, page 110. It will be seen that all stars between P and E (X for example), although having 18 hours' right ascension, will be carried by the apparent rotation of the sphere round E in a direction from right to left; therefore these stars will appear to move from X to X′, and will therefore decrease their right ascension from year to year, instead of increasing it. The stars ε Ursæ Minoris, δ Ursæ Minoris, λ Ursæ Minoris (see *Nautical Almanac*), are good examples of stars which thus change their right ascensions by decreasing them annually, and to a large amount, as they are near the pole of the heavens.

As stars between P and E decrease their right ascensions by this apparent rotation of the sphere, whilst stars situated at A and O increase their right ascensions from the same cause, it is evident that there will be several points in the heavens where stars will not change their right ascensions at all, but only their north polar distance. These points will be those where an arc joining the star or point with E is at right angles to an arc joining the star or point with P. Such points would be at Z and Y. At these points the stars will be moving either exactly towards or exactly away from P, and consequently would give no change in the hour angle at the pole by their change of position.

There is a law connected with this fact which we

cannot find has hitherto been noticed by astronomers or geometricians; and as it shows very easily where those points are located, and will enable us to understand better the cause of the so-called 'proper motion of the fixed stars,' we will here describe what this law is.

Let E, diagram below, represent the pole of the ecliptic, assumed to be the centre of the circle traced



by the earth's axis during one revolution of the equinoxes. Let P be the pole of the heavens. Let X, Y, Z represent three concentric circles, E being the common centre. Let these circles represent the course which certain stars would follow round E during an entire revolution of the equinoxes. These stars would appear to move round their respective circles in the direction shown by the arrows.

We have stated that there are various points in the heavens which are so situated that a star, when localised on these points, will not change its right ascension, and we will now demonstrate where these points are for various stars.

Join P E and bisect P E at A. With A as a centre and radius, A E or A P describe a circle on the sphere. Call this circle S P L E. Where the circle S P L E intersects the various circles X, Y, Z, &c., at these intersections will be the points where the various stars moving over these circles do not change their right ascension. Thus the star whose colatitude is E X will not change its right ascension when at O or at L; the star whose colatitude is E Y will not change its right ascension when at R or at M, and so on. It will be seen that there are only a few stars that are so privileged, as we may term it. These points, however, there is no mistaking; they all lie on the circumference of the circle SPLE; and when we know the colatitude of any star, we can at once calculate when it will occur that this star's right ascension shall not vary.

Take, for example, the star δ Ursæ Minoris, the AR of which is about $18^h\ 14^m$, and its north polar distance in 1872 about $3°\ 24'$ (omitting seconds). The colatitude of this star may be taken in round numbers as $90° - 69°\ 58' = 20°\ 2'$. Now this star will not change its right ascension when an arc from the star to E is at right angles to an arc from the star to P.

Let the circle X O L be the circle supposed to be traced by δ Ursæ Minoris, it is required to find where

the point L is located at which this star will not change its right ascension.

We have, in the spherical triangle P L E, P E $=23^\circ\ 27'\ 24''$, E L $=20^\circ\ 2'$, the angle E L P a right angle, to find P L, the polar distance, and E P L (the right ascension of L short of 270°).

Upon working out this case, we obtain $59^\circ\ 23'\ 3''$ for the angle E P L, and $12^\circ\ 27'\ 41''$ for the arc or polar distance P L.

Converting these items into right ascension and declination, we obtain for the point L a right ascension of $14^h\ 2^m\ 28^s$, and a declination north of $77^\circ\ 32'\ 19''$. Thus any star exactly at that point in the heavens will not vary its right ascension, in consequence of the polar movement; but it will not vary its AR at that particular point only, provided the pole of the ecliptic is the real centre of polar motion. The star 4 Ursæ Minoris is near this point, and as will be seen on examining any good modern catalogue, varies its right ascension very slightly, viz. $0^s{\cdot}387$ annually; whereas if it were exactly at the point L, it would not vary its right ascension at all.

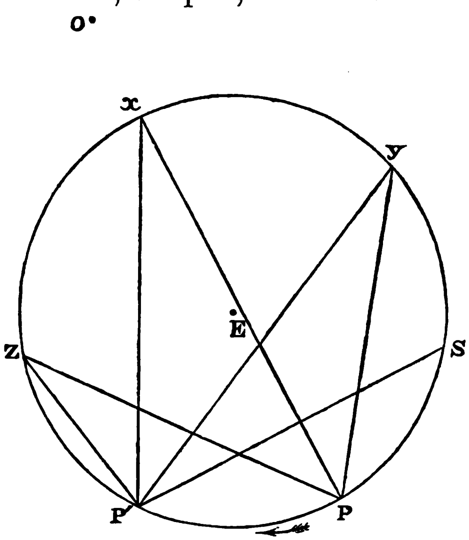
From the data given above, it will be easy for any person to calculate or determine those points in the heavens which are located on the circumference of the circle S P L E, and thus to determine where a star's right ascension will not vary more than a very minute quantity during several years.

We will now describe another law relative to changes in right ascension, which, when thoroughly

examined, leads to very interesting results. It is with reference to the change of right ascension between any two stars.

In order to demonstrate this law, we must consider that the pole of the heavens now moves, and not the stars, the pole of the heavens being still supposed to move round the circumference of a circle the centre of which is the pole of the ecliptic.

Let E, diagram below, be the pole of the ecliptic, supposed to be the centre of polar motion. Let P be the pole of the heavens, and let Z, X, Y, S, be four stars or points in the heavens, just as far in angular distance from E as P, the pole, is from E.



The difference in right ascension between Z and S will be measured by the angle Z P S. The difference in right ascension between X and Y will be measured by the angle X P Y.

Let the pole P move to P′ round E as a centre. The difference in right ascension between X and Y will now be measured by the angle X P′ Y; and the angle X P′ Y is equal to the angle X P Y, because P has moved round the circumference of the circle of which X Y is a chord. Thus these two stars have not changed their *relative* right ascensions. Hence each star must have increased its right ascension equally during the movement of the pole from P to P′. In the same manner, the angle Z P′ S is equal to Z P S; and the stars Z and S have not altered their *relative* right ascensions during the movement of the pole from P to P′. Hence we obtain the following law, which is most important in its results:

The relative right ascension of any two points on the circumference of the circle traced by the pole of the heavens never varies.

The facts deducible from this law are numerous, important, and interesting, thus:

Let S be a point on the circumference of the circle traced by the pole, P′ another point on the circle. Now the angle formed at P by S and P′ will be a constant; therefore the angle P′ P E will increase just as much as E P S decreases by the movement of the pole from P towards P′. In like manner, the angle X P Y will decrease, just as much as Z P X increases, by the polar movement towards P′.

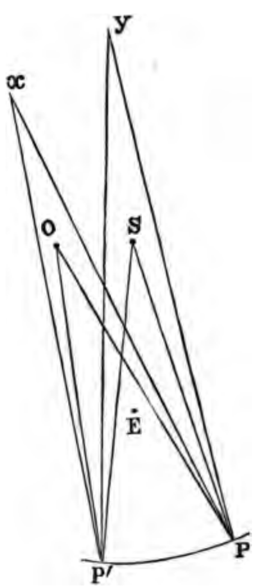
Again, let the angle Z P S be 90°; then if, during the polar movement towards P′, the angle at P remain constant at 90°, then we know the course of the pole

in the heavens is on the arc of a circle the circumference of which passes through Z and S.

Again, let us assume that there is a star on the circle described by the pole, and situated as is the star Y (see diagram below), and that there is another star, O, that passes the meridian at the same time as does X, when the pole is at P.

When the pole was located at P′, the difference in right ascension between X and Y would be just the same as it was before; but the difference in right ascension between O and Y would be greater than it was when the pole was at P, because the angle O P′ Y is greater than X P′ Y.

Let us now take another case. Suppose, instead of having the whole circle traced out for us, we have only a small arc of say 25° traced by the pole, viz. from P to P′; and let us suppose that we assumed E to be the centre of the circle of which we supposed P P′ was part of the circumference.



Let X and Y be two stars which would be supposed outside the circle of which P P′ is part of the circumference, because E Y and E X are both greater than E P. (These two points are in reality on the circumference of the

circle of which P P′ is an arc.) Now the difference in right ascension between X and Y can be measured by the angle at the pole X P Y; and the difference of right ascension between O and S is measured by the angle O P S.

When the pole has moved to P′, the angle X P′ Y will be exactly the same as X P Y, because P′ P X and Y are on the circumference of the same circle; but the angle O P′ S will not be the same as O P S, because S and O are not on the circumference of the same circle as P′ P. Hence it follows that, whilst X and Y have changed their right ascension uniformly, O and S have not changed their right ascension uniformly. But on the assumption that E is the centre of the circle described by P, and that P, P′, O, and S, being equidistant from E, are on the circumference of this circle, O and S ought to have changed their right ascensions uniformly, whilst X and Y ought not to have done so.

What, then, are we to say? Are we to announce that all these four stars have a proper motion in right ascension, because we have assumed for the pole an erroneous point as a centre round which it is supposed to travel? We certainly find a marked difference in the relative right ascensions; and this is only another way of announcing a different annual rate in the right ascensions of the two stars, according as O and S or X and Y are on the circumference of the same circle as are P and P′.

Hence it follows, as an absolute law, that we cannot

calculate the right ascension of any star for any date in the future, unless we know the exact relative position of this star as regards the circle described by the pole of the heavens.

Hence also, unless we know the exact value of the radius of the circle described by the pole of the heavens, and the exact point at which the centre of this circle is located, we cannot predict the right ascension of any star for a future date.

There are many other valuable problems deducible from the preceding, some of which we will now enter upon. From an examination of the preceding demonstrations it will be evident that, if we committed the mistake of attributing either to the stars X and Y, or O and S, a proper motion in right ascension because they varied their right ascensions in a manner not in accordance with our suppositions connected with the polar movement, we should be laying the foundation for a confusion that would almost defy correction.

It will be evident from even the preceding demonstrations that it is impossible for any computer to know what ought to be the actual change produced in the right ascension of any star, no matter where this star was situated, unless he knew the value of the radius of the circle traced by the pole of the heavens. In the examples we have given in the preceding pages, we have, for the sake of illustration, treated the pole of the ecliptic as the centre of the circle that the earth's axis really traces in the heavens, and the general principles relative to the changes

in the right ascension of the various stars have been explained therefrom. But both in the earlier pages of this work, and more fully in our last work—viz. *The Cause, Date, and Duration of the last Glacial Epoch*—we pointed out and demonstrated that the pole of the ecliptic was not the centre of the circle traced by the earth's axis, but was 6° from the centre; and that the circle which the pole of the heavens described was not one having, as had hitherto been erroneously assumed, a radius of 23° 28′, but one having a radius of 29°.25′ 47″.

The preceding demonstrations, however, will all hold good in principle; and we have merely to substitute the true centre of the circle described by the pole for the pole of the ecliptic in these diagrams, in order to note what results occur as regards the changes in stellar right ascension.

Hence any two stars on the circumference of the actual circle traced by the poles of the heavens will not vary their relative right ascensions. Also those stars which are so situated that the arc from the centre of polar motion to the star is at right angles to the arc from the pole of the heavens to the star, will not at that date change their right ascensions, or the change will be so slight for a short time as to be almost imperceptible.

Now, considering that there is at one point in the heavens no less than 12° difference between the circumference of the circle actually traced by the pole and the circumference of the circle of which the pole

of the ecliptic is the centre, and the radius of which is 23° 28′, it will be evident that most noticeable differences will be manifest in the results which really occur from those which would be supposed to occur, on the erroneous assumption that the pole of the ecliptic was the true centre of the circle traced by the earth's axis, and that the radius of this circle was 23° 28′.

It is from this law that the cause of the so-called proper motion of the fixed stars will meet its solution. Hitherto the pole of the ecliptic has been supposed the centre of the polar circle, whereas it is not so; and the supposed changes in stars' right ascension have been calculated on this erroneous supposition. The difference between their actual change and supposed change has been attributed erroneously to proper motion, as will be demonstrated in future pages.

CHAPTER IX.

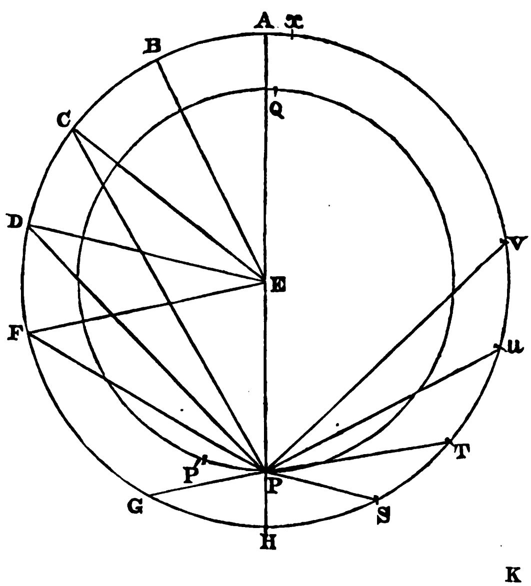
GEOMETRICAL LAWS CONTINUED, AND THE DEMONSTRATION OF THE CHANGES OCCURRING AT THE SO-CALLED APEX OF SOLAR MOTION.

WE must now refer to another law connected with the changes in both right ascension and declination, and it is a law which is very easily demonstrated if we select some stars; but although not so apparent if we choose others for demonstration, it yet holds good with regard to every star in the heavens, no matter where it may be situated. This law produces the most marked results when we examine two star-catalogues separated by a long interval of time. Referring to diagram, page 117, we will endeavour to show how easily this law is demonstrated as regards a star moving over a circle such as X O L; it also holds good with regard to the course of every other star in the heavens, no matter whether within or without the circle described by the pole. This law is, that the star in its curve varies *the rate* at which it changes its annual right ascension. Thus at one period a star's annual rate may be say 4^s annually, and at another period, say 20 years before, the star's annual rate of change in right ascension was 3^s annually,

then it is not true that midway between these dates the star's annual rate was $3^{s} \cdot 5$. A very clear proof of this law may be given by referring to diagram, p. 117, where a star at O will, from what has been already stated, be found not to change its annual right ascension. Again, if we examined it *n* years afterwards, when it was at L, and found it still had no annual rate of change in right ascension, it is evident that if we assumed that midway between the two dates it also had no change in right ascension, we should be making a most palpable error. Also, if we found the annual rate of change in right ascension when the star was midway between O and L, and then found at L no rate at all, we could not with accuracy state that midway between these two dates the star's rate was a mean between nothing and its rate when first noted. In fact, it is a law that a curve does not change its distance from a fixed point uniformly, unless the curve be tracing an arc exactly towards the point, which is theoretically impossible. Thus no star in the heavens can have its mean rate accurately found by taking the mean of its rates at two intervals. There is in this geometrical problem a 'second difference' to take into account, and sometimes even a third difference, just as there is in connection with a lunar distance. For this is a very similar case to that of the moon, which in equal intervals of time does not change its angular distance uniformly from a star. There are what we may term three divisions in each hemisphere, in which

divisions the changes in right ascension and declination due to precession are different. Under the first division are stars whose colatitude is greater than the radius of polar motion. Now, taking the obliquity as about 23° 27′ 30″, we should in this division take a great number of stars which were circumpolar, and a great number which were not circumpolar. Any stars having about 18 hours' AR and a greater colatitude than 30° would not be circumpolar, for the latitude of Greenwich. Thus in this first example we take stars of two kinds, viz. circumpolar and those that are not circumpolar, yet the same law holds good with all stars whose colatitudes are greater than the radius of the circle traced by the earth's axis.

To illustrate this law, we will refer to the following diagram.



Let E be the pole of the ecliptic, supposed to be the centre of polar motion; P the pole of the heavens. Let A B C D F be the supposed course of a star round E, which supposed course would be similar in its effects to the pole P moving round E in the direction P P′. Suppose P E the meridian of 18 hours' AR. Then the star, as it appeared to move over uniform arcs in uniform intervals of time, would increase its right ascension, but would not increase it at a uniform rate.

Thus let the arcs A B, B C, C D, D F, be all equal, and equal also to G H. Then the difference between the angles B P A and B P C would not be the same as between B P C and C P D; for as this star approached the pole, its rate of variation in right ascension would increase.

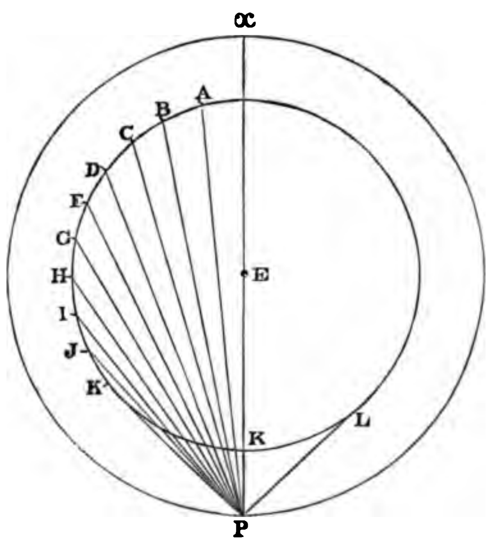
If this law be traced out, it will be found that for all stars whose colatitudes are greater than the radius of polar motion, and whose right ascensions are between 18 hours and 0 hours, and 0 hours and 6 hours, their annual rate of change increases, the cause for which will be seen by noting how the angle A P B in time becomes G P H. For if the pole move round to Q the effects would be the same as if the star moved round to G and H.

We will next take a star moving round from S to T, U and V. This star would increase its right ascension, but its rate would be a decreasing one, because the difference between the angles S P T and T P U is greater than the difference between the angles T P U

and U P V. If this law be traced out, it will be found that the annual rate of variation in right ascension of all stars whose colatitudes are greater than the radius of polar motion, and whose right ascension is between 6 hours and 12 hours, and 12 hours and 18 hours, is decreasing. Thus the rate is a variable, and is + from 18 hours to 6 hours, and — from 6 hours to 18 hours.

The change in the rate of variation of north polar distance is slightly more complicated than is the change in rate of the AR. The polar distance of all stars between 18 hours and 6 hours will decrease, and the rate of decrease will be as follows:

Let P A, diagram below, be the polar distance of the



star when at A, P B its polar distance when at B, and so on. It will be found that the variation in the length

of the arcs P A, P B, &c. is dependent not only on the angular distance of the star from P, but also its angular distance from the centre (E) of polar motion.

We will next explain the changes in right ascension which will occur to a star within the limits assigned, and whose angular distance from the centre of polar motion is less than the radius of polar motion.

Referring to diagram, p. 131, let P be the pole of the heavens, E the pole of the ecliptic (assumed as the centre of polar motion), consequently P E X a meridian of 18 hours' AR. Let A B C D, &c. represent the course of a star round E as produced by precession. The angles A P B, B P C, &c. representing the amount of change in right ascension due to a constant movement in arc of A B, B C, &c. The rate of variation in this case will be decreasing as long as the star's right ascension increases; when the star reaches a point I, where E I P is 90°, the star's right ascension will decrease and its rate will increase. On reaching K the star's increase of rate will be nearly constant; it will then decrease its rate until it reaches a point L, where E L P is 90°.

On passing the point L, this star, which decreased its right ascension from I to L, will again increase it; and as it increases its right ascension it will also increase the rate of its increase.

Hence all stars thus situated have, during a revolution of the equinoxes, four marked changes; so that four circumpolar stars, having but slightly different right ascensions, and but slightly different north

polar distances, would vary their right ascensions and the rates of their variations in very different ways.

That which must be particularly noted is, that a star *within* the circle traced by the earth's axis varies its rate in the opposite direction, for a great part of its course, to that in which a star outside the circle varies its rate; thus we must know when a star is within and when without the polar circle in order to know what the variation in its rate should be.

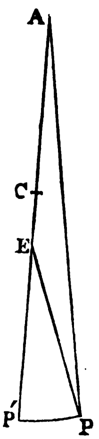
In the polar distance there is also a marked difference in the changes as regards stars within and without the circle traced by the earth's axis. The greatest change in the polar distance will be when the change in AR is least; thus at I and L a star's change in polar distance will be greater than at any other part of its apparent course. The rate of the change will vary most when the star is midway between P E and P I, and P E and P L. Thus there will be four maxima and four minima changes in the rate.

All stars that are situated within the circle traced by the pole will be affected in this manner, and stars thus situated may be any whose north polar distance is equal to twice the radius of the polar circle, and in the direction found by producing the arc joining P and the centre of this circle to the length of the diameter, and those stars that lie within this circle. The stars thus found will be circumpolar stars, varying from about 13 hours' AR near the pole, round to 18 hours, and with a north polar distance of

twice the radius of polar-motion, and on to about 23 hours' AR for those stars near the pole.

In order to find the right ascension and north polar distance of this point for 1873, we have the following data:

E represents the position of the pole of the ecliptic, P′ the position of the pole at the date 2295·5 A.D., C the centre of polar motion, C A = C P′ = 29° 25′ 47″, C E = 6°.·. A E = 35° 25′ 47″. The angle P E P′ = angle of precession between 2295·5 A.D. and 1873, which, at 50″·3 per annum, amounts to about 5° 54′. The angle P E A = 180° − 5° 54′ = 174° 6′. We have, then, P E and A E, that is, the obliquity 1873 and A E, and the included angle at E, to find P A and the angle E P A. From this data the angle E P A will be found about 4° 56′, and the side P A about 58° 53′. As the right ascension of E is 270°, the right ascension of A is 265° 4′. Consequently the right ascension and north polar distance of the point A, at which these changes in stars' rates vary, is 265° 4′ right ascension and 58° 53′ north polar distance.



The point thus found we will again refer to, as most important results depend thereon;* for it is that point in the heavens at which it has been found by

* The attention of astronomers is called to this problem. The point so long called the apex of solar motion, and supposed to be that point towards which the whole solar system was travelling, is that point in the true circle described by the pole farthest removed from the pole of the ecliptic.

modern computers that stars appear to separate, when their theoretical right ascensions are compared with their actual right ascensions. It is the point termed *the apex of solar motion* by modern theorists.

In an investigation of an original nature like the present, we prefer incurring the risk of unnecessary repetition rather than to leave a subject obscure in consequence of incomplete demonstration. We will therefore once more refer to the principal changes produced on the right ascension of stars, and due to the movement of the pole of the heavens.

First, those stars which are situated exactly in that part of the heavens where an arc from the centre of polar motion to the star, is at right angles to an arc from the star to the pole, will not change their right ascension when so situated. Consequently we must know exactly the position of the true centre of polar motion in order to know when a star is so situated. It follows from the above, that two stars might be within half a degree of each other, one star being situated exactly so as not to change its right ascension, the other, close to it, would, however, change its right ascension considerably. It would be impossible for any computer or observer to know whether either or both of these stars' changes in right ascension, found by observation, was due to the movement of the pole alone, unless he knew exactly the position of the centre of polar motion, and consequently knew exactly the relative position of the stars as regarded the circumference of the circle traced by the pole of the

heavens. Endless positions might be marked on the sphere of the heavens where a star's change in right ascension might be exactly what it should be if the centre of polar motion were 6° from the pole of the ecliptic; whilst exactly the same change in the same star's right ascension would indicate that the star itself had an independent movement of its own, if the pole of the ecliptic really were the true centre of polar motion.

Secondly, the relative right ascensions of any two stars located on the circumference of the circle traced by the pole of the heavens will not vary. This law having been recognised, it follows that if we find that the relative right ascension of two stars does not vary, then they change their *actual* right ascension just as they ought to do, provided they are on the circumference of the true circle traced by the pole; but if they are not on the circumference of that circle, and yet do not vary their relative right ascensions, then it follows they must have some independent motion of their own. In order, then, to know whether or not they have an independent motion of their own, we must know where is the centre and what is the angular value of the radius of the circle traced by the earth's axis in the heavens; for unless we do know this preliminary but all-important fact, it is a mere scientific absurdity to assert that a star has an independent motion of its own, because it varies its right ascension in a slightly different manner from what it would do supposing the pole of the ecliptic

were the centre of the circle traced by the earth's axis.

Thirdly, it being a fact that the change in the position of the pole of the heavens is due to the change in direction of the earth's axis, and that the earth's axis cannot change its direction without the whole earth partaking of this movement, it follows that the actual movement of the earth, of every zenith, and of every meridian, will be different according as the earth's axis traces a circle round one or another point in the heavens. As the right ascensions of stars are registered according to the time and manner in which they transit a terrestrial meridian, it follows that, unless we know correctly how a terrestrial meridian ought to vary, we cannot know whether certain stars transit this meridian as they ought to do, or whether they have some independent motion of their own. Consequently it follows that, even to know how our meridian ought to change, we must at least know where the centre of polar motion is located. Besides this one important fact, we must also know the actual movement of the sphere which accompanies the change in direction of the earth's axis.

This movement of the sphere which accompanies the change in direction of the earth's axis will form a special subject of investigation in this work, for it is one which, in its geometrical results, does not appear to have been as yet treated in that searching manner which so important a problem demands.

Without, however, at present referring farther to the detail changes of any given meridian, we will merely request that the reader's attention be directed to the importance of knowing where the centre of polar motion is located, and what is the radius of polar motion, before it is possible to know or to calculate what the changes in a star's right ascension ought to be, and before therefore it is possible to state that a star's change in right ascension is such as to indicate that the star has an independent motion of its own.

A careful study of the preceding problems will enable the reader to understand the next chapter.

CHAPTER X.

THE CAUSE OF THE SO-CALLED PROPER MOTION OF THE FIXED STARS.

THE so-called proper motion of the fixed stars is a problem which, for upwards of a hundred years, has occupied the attention of astronomers. To the individual unacquainted with technical astronomy the term proper motion of the fixed stars gives an idea which is not correct. To such a person this term seems to imply that various stars are actually altering their angular distance from each other, and that this alteration, if continued, and if exaggerated in amount, would actually cause the shape, as we may term it, of a constellation to alter. Such is not the case. The only suitable instrument by which we can measure the actual angular distance between any two stars is the sextant. The best sextants read only to 10″. There is consequently no possibility of obtaining a more accurate result than 10″ in the angular distance of two stars with a sextant; and as any two stars would, in consequence of refraction, be each raised towards the zenith, and consequently brought nearer to each other than they really were, any results obtained by sextant measurements have been deservedly rejected.

The instruments which for many years have been considered the most dependable in all observations are the transit instrument, the equatorial and the mural circle. By each of these instruments the right ascensions of stars have been determined, and *it is from a comparison of the right ascension of various stars at different dates that it has been concluded that the stars have a proper motion of their own.*

The reader who has followed us in the demonstrations we have given in the last chapter will at once perceive, that when any observer records the changes that occur in a star's right ascension and declination, and attributes a portion of this change to precession and a portion to the actual motion of the star itself, he is at once venturing on most dangerous ground. Before he can state what is the amount of proper motion in a star, he must be able to define with minute accuracy what ought to be the actual change in right ascension produced by the movement of the pole of the heavens. Before he can define what ought to be the actual change in right ascension of a star due to the movement of the pole, he must be able to define the exact position of the centre of the circle described by the pole, the exact radius of this circle, the exact changes of the earth combining with and due to this change in the direction of the earth's axis, and also be able to define whether the centre of polar motion is a fixed or a variable point in the heavens.

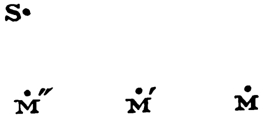
The actual conditions, however, are, that the

olden astronomers were not acquainted with, or overlooked the fact, that the pole of the ecliptic was not the centre of the circle traced by the earth's axis. They were therefore unacquainted with the true position of the centre of polar motion, and were also unacquainted with the value of the radius of polar motion, and of the true course of the pole of the heavens as regards the fixed stars.

If these former observers had adopted a sound method of discovering what were the actual differences in the right ascension of stars when their theories and observed facts were compared, they would yet have obtained only confusion as long as they had remained unacquainted with the true course of the pole of the heavens. When, however, we find, on examining their records, that they adopted a system unsound in principle, for discovering what they thought was a proper motion, we can easily perceive that a mass of confusion must arise, which it is not surprising has defied solution during the past 100 years.

In the latter part of the last chapter we called particular attention to the fact that a star, no matter in what position relative to the pole of the heavens or to the centre of polar motion, did not maintain a uniform rate in its increase or decrease of right ascension. This *variation in the rate* is a most important item; for this variation not being uniform, it follows that the rate at a date midway between two extreme dates will not be the mean of the rates at

the extreme dates. Problems of this nature are of frequent occurrence in practical astronomy, and we will give two illustrations, one of which is as regards the decrease of distance of the moon from a star. Thus, suppose S a star, and M the moon, moving in the direction M M″. Suppose the moon at M 13° from S, and approaching S at the rate of 30′ per hour. At M″ we will suppose the moon 2° from S, and not altering its distance from S at the instant of observation. It would then be incorrect to state that midway between M and M″ the moon was decreasing its rate from S 15′ per hour. And this principle, as we have shown in the last chapter, holds good for the changes, both in right ascension and declination, of every fixed star.



Again, we find that on the 21st of March the change in the sun's declination per hour is about 59″·29, and on the 20th of June it is about 0″·24. About midway between these dates would be the 5th of May; but we do not find that on the 5th of May the sun's hourly change in declination is

$$\frac{59''\cdot29 + 0''\cdot24}{2} = 29''\cdot76$$

but it is in reality about 42″ per hour.

When, then, we find that the earlier astronomers, who published a list of what they supposed were proper motions of fixed stars, did not appear to have considered the variable results which must follow the assumption of an erroneous centre of polar mo-

tion, we must perceive how undependable are such conclusions. When, however, we find, in addition, that the method recommended by the olden astronomers (and followed with but trifling alterations by more modern observers) is one that assumes the rate in a star's change of right ascension is uniform, instead of, as it is, a variable, this mystery of the so-called proper motion of the fixed stars is perhaps one of the most remarkable instances of long-continued confusion ever seen in the annals of science.

The first paper to which we shall refer is one that has been often quoted by modern writers as a sound and most valuable paper, and although some modifications of the method therein recommended have been adopted by later observers, the general principle herein described has been followed. This paper may be found in vol. v. *Memoirs of the Royal Astronomical Society*, under the title of 'Proper Motion of 314 Stars.' The writer in this paper states:

'The most satisfactory mode of determining the proper motion of a fixed star is by a comparison of two catalogues of distant epochs; for if the difference in its position at the two epochs does not correspond with the amount of the precession of the equinoxes, there is good reason to suspect that the star has a proper motion of its own, particularly if the difference exceed what may fairly be attributed to the errors of observation.'

In this statement the reader who has followed

the demonstration in the preceding chapters will at once perceive a grave error. The writer of the above recommendation assumed that the pole of the ecliptic was the centre of polar motion, and that the change in right ascension of a star corresponded exactly to the precession of the equinoxes. He was not aware that, granting that the centre of polar motion was even 50° from the pole of the heavens, the precession of the equinoxes would be almost identical with what they would be if the pole of the ecliptic were the centre of polar motion; but the changes in the star's right ascension would be very different if the centre of polar motion were 50° from the pole from what it would be if the pole of the ecliptic were the centre. But this error, grave as it is, is small compared to the next in the same paper, for the writer goes on to say:

'In the former of these [catalogues] the stars are reduced to the beginning of the year 1755, and in the latter to the year 1800, and by means of the annual precessions annexed by M. Bessel to each star in Bradley's Catalogue for the two epochs, we may determine with great accuracy how much a star is effected by precession alone in any given period; whence the difference (if any, and exceeding the probable errors of observations, &c.) may be fairly attributed to proper motion.

'Thus, if P denote the place of a star in Piazzi's Catalogue (either in right ascension or declination), and B the place of the same star in Bradley's Cata-

logue, p the annual precession in 1800, and π the annual precession in 1755, both taken from M. Bessel's Catalogue in the *Fund. Astron.* above referred to, the annual proper motion, μ', of a star for the *first period* (viz. 1755 to 1800) will be

$$\mu' = \frac{P-B}{45} - \frac{p+\pi}{2}.$$

And it is in this manner that I have deduced the annual proper motion of the stars in the list subjoined to this memoir.'

It is scarcely necessary to point out, that the method here described as that adopted by the writer of the paper quoted, is unsound. The method here recommended, and supposed to give the proper motion of the fixed stars, in reality gives only the difference between the mean rate, and the total amount of the rate, divided by the number of years during which the rate is allowed for. If a star's rate of right ascension varied uniformly, then the above method would give us some chance of finding the proper motion, if it existed, but only on the condition of our having previously ascertained the true centre and radius of the circle traced by the earth's axis on the sphere of the heavens. As, however, the rate is not only not uniform, but does not vary in a uniform manner, it follows that the discordances obtained by the differences between

$$\frac{P-B}{45} \quad \text{and} \quad \frac{p+\pi}{2}.$$

will not be proper motion in the stars, but the difference

between the total change in right ascension divided by 45, and the mean of the rates at the two intervals. It seems singular to find so very elementary a mistake in principle, in a work so carefully criticised as the *Memoirs of the Royal Astronomical Society*, and equally as remarkable that the results obtained from this process, are still quoted as undeniable proof that the various stars have a proper motion of their own.

At another portion of the same article on the proper motion of 314 stars, the writer states, that when a star was within 10° of the pole he adopted another method for determining the proper motion. This method was the following:

'Suppose α to denote the right ascension and δ the declination of a star in one catalogue, and α' its required right ascension in the other catalogue, then

$$A = \alpha + 11'\ 28''\cdot 04 \qquad n = \cdot 0029176 \times \tan.\ \delta$$

$$\tan.\ x = \frac{n \sin.\ A}{1 - n \cos.\ A} \qquad \text{whence } \alpha' = \alpha + 23'\ 1''\cdot 45 + x.\text{'}$$

Both this and the former system every intelligent geometrician will perceive are unsound. They both omit to notice that the pole of the ecliptic is not the centre of polar motion; and at the time when the writer referred to, recommended his process, there was not the slightest idea of the centre of the circle traced by the pole of the heavens being as much as 6° from the pole of the ecliptic.

In order to make this problem as clear and intelligible to the reader as it ought to be, we will prac-

tically test the method recommended by the writer of the above article, and note to what results we are led.

Referring to diagram on page 117, we find a star S, owing to the movement of the pole round E, appearing to move round E, whilst the pole appears stationary; that is, transferring the real movement of the pole to an apparent movement in the stars, which would produce the same effects. Now a star that appeared to move round the circle Z S N in consequence of the movement of the pole of the heavens we will take as one so near the pole of the ecliptic as to be always more than 10° from the pole P. This star we will suppose to be located at S at the date of one catalogue of stars, and consequently to have at that time a right ascension of 270° + the angle E P S. The annual change in right ascension of this star when at S would be nothing, because the star is then moving apparently directly towards the pole P, and consequently is just in that position where, as has been already shown, a star does not change its right ascension for a brief period. We will next suppose this star to have been carried round to N, owing to the apparent rotation of the heavens, and due to the movement of the pole of the heavens, and to be located at N at the date of a second catalogue. The right ascension of the star N would now be 270° − E P N, and its rate would be 0^m per annum, in consequence of its being situated exactly in that portion of its apparent orbit where it was moving directly away from the pole.

At the date of both catalogues, therefore, the star would have no annual rate of right ascension. At the date of the first catalogue the star had a right ascension of $270° + \text{E P S}$; at the date of the second catalogue, $270° - \text{E P N}$. Let us take $\text{E P S} = \text{E P N} = 12°$. Then the right ascension of the star at the date of the first catalogue would have been 282°; at the date of the second, 258°.

Now let us apply the method mentioned above as that best suited to give us the proper motion of stars, and let us suppose θ as the years between the two catalogues.

Substituting the values above in the formula recommended, we have

$$\frac{282° - 258°}{\theta} - \frac{0 - 0}{2} = \frac{24}{\theta}$$

for the proper motion of the star; that is, a star which really has no proper motion, but merely is affected by the change in direction of the axis, has assigned to it in θ years a proper motion of 24°.

In the example just quoted we have taken a star in such a situation as to give the greatest errors arising from the principle recommended, but every star, wherever situated, would give similar results, though in a less degree; for the principle is unsound, and the formula given does not enable a computer to obtain the proper motion of the fixed stars, but merely the difference between the mean rate of the star's change in right ascension, and the total change,

divided by the number of years during which the change has been observed.

If a star happened to change its right ascension annually at the date 1700 to the amount of 8 seconds, and at the date 1800 changed it annually at the rate of 10 seconds, it would not be true that the star's rate per annum at 1750 would be exactly 9 seconds. It is needless to point out to any sound geometrician that such a result as taking a mean at the date midway between the extreme dates, and assuming this to have been the mean annual rate, must lead to incorrect results. Yet this is the system which, with but little variation, has been adopted, and which was supposed to give the actual motion of the fixed stars. And this system was supposed to reveal a proper motion, when it was not known that the actual course of the pole of the heavens was such as to render it impossible that the pole of the ecliptic could be the centre of the circle traced by the pole of the heavens, and the true position of this centre was unknown, and the actual curve traced by the pole must also have been unknown.

These facts show that geometry is at present a science so neglected by many astronomers that they commit the gravest errors unconsciously, and lead themselves and their followers into errors which result in endless complications and confusions.

The list of stars supposed to have a proper motion, and given in vol. v. *Memoirs of the Royal Astro-*

nomical Society, is valueless, as the results are obtained by employing a method which is not correct, and omitting to make corrections or allowances for other items which must be taken into account before we can tell whether a star's change in right ascension is, or is not, due entirely to the movement of the pole of the heavens, and to the resulting movement in the earth itself.

The method adopted to determine the so-called proper motion of the stars in vol. xix. *Memoirs* differs but slightly from that adopted in Mr. Baily's list. The proper motion in all cases is obtained by subtracting the mean annual precession from the mean annual observed motion. But at the very outset the geometrician will observe that we have an uncertain item, viz. *What is* the mean annual precession of any star, and how can it be obtained?

As the apparent change in the position of a star relative to the pole and a given meridian is due wholly to the change in direction of the earth's axis, which causes the pole itself to alter its position, and as this change is almost similar in its effects to that which would occur if the stars revolved round that same point round which the pole travels, whilst the pole remained fixed, it follows, as a law from which there is no escape, that unless we can tell where the point is located round which the stars do appear to revolve, we cannot tell what ought to be the annual precession either in right ascension or declination of any star. Consequently we cannot tell what the

proper motion is, or even whether the star has any proper motion at all.

Elaborate and exhaustive in some respects as are the preliminary calculations and the details of those papers referring to stars' supposed proper motions, yet we may search in vain for any reference to that all-important item, viz. the exact course of the pole of the heavens, and the exact position of the centre of the circle described by the pole. Without such knowledge the whole of the calculations based on the assumption that the pole of the ecliptic is the centre of the circle are valueless.

Most able papers on the supposed movement of the solar system in space, as demonstrated by the imagined proper motion of the fixed stars, have been published by several astronomers of note. Among these are the papers of the Astronomer Royal, Mr. Stone, Mr. Dunkin, and others. There is, however, in all these papers a defect exactly similar to that which exists in the theoretical conclusions of those mathematicians who explained the reasons *why* the pole of the heavens described a circle round the pole of the ecliptic as a centre.

Former speculators assumed as a basis that the pole of the heavens traced a circle round the pole of the ecliptic as a centre, and never varied its distance from this centre, and they then worked-out elaborate theories to explain why it did so. That it did not do so was a fact unknown to them, but it is one somewhat likely to diminish the value of their theories.

So is it with the supposed proper motion of the fixed stars, and the supposed movement of the solar system in space towards that point in the heavens defined as the apex of solar motion. It has been imagined that the discordances between the actual right ascension and declination of a star and the right ascension and declination it ought to have, *on the supposition that the pole of the heavens traced a circle round the pole of the ecliptic as a centre*, were alone due to the proper motion of each star. It is unnecessary to again point out that such a conclusion is incorrect.

From what has been demonstrated in chap. viii., the reader will understand that the rate at which the right ascension of a star varies is a variable quantity. The rate of change in the right ascension we showed changed four times during a whole revolution of a star round the centre of polar motion, provided this star was always located within the circle traced by the pole; we also showed that at nearly the opposite point in the circumference of the circle traced by the pole, to that point at which the pole was situated, was *the* point at which the rate of change in right ascension altered from + to −. Thus, referring to diagram on p. 134, this point of change would be near the point marked A; it would, in reality, be that point in the circumference of the real circle traced by the pole of the heavens farthest removed from the pole of the ecliptic. To find where such a point is we must produce the arc joining the pole of the ecliptic with the centre of polar motion,

and where this arc cuts the true polar circle, there will be the point on either side of which the rate of change in the right ascension of stars varies.

To ascertain where this point is situated is not difficult. Referring to diagram, p. 134, E represents the pole of the ecliptic, C the centre of the circle described by the pole, P, P′, A, three points on the circle of which C is the centre, P′ the position of the pole at the date 2295·5 A.D., P the position of the pole at the date 1870; consequently P P′ represents the arc traced by the pole between 1870 and 2295·5 A.D. C A is a known quantity, viz. 29° 25′ 47″; C E is 6°, therefore E A is 35° 25′ 47″; E P is the obliquity at the date 1870.

From the above data we can find the value of the angle E P A and the arc P A.

The right ascension of A at the date 1873 will be 270°—E P A, whilst the north polar distance of A will be P A. Upon calculating these values, it will be found that, in round numbers, the right ascension of A at the date 1873 was about 265° 4′, and the north polar distance of A was about 58° 53′.

When it was supposed by the earlier astronomers that the unaccountable discordances found to occur in the right ascensions and declinations of stars were to be accounted for only on the assumption that the stars themselves had an independent motion of their own, an investigation was made of their observed variations, and it was found that there was a point in the heavens at which the stars appeared to separate, some appearing to have a motion to the right, others

to the left of this point. The conclusion arrived at from these supposed changes was, that the sun and the solar system were moving towards this point in the heavens, and thus caused the discordances in right ascensions that were hitherto inexplicable. In order to account for the discordances, it was found necessary to assume that the solar system was travelling towards this point in the heavens at a rate of no less than 154,000,000 miles per year; and that consequently, during even the Christian era, the system must have approached this point, if the motion were uniform, by no less than about 300,000,000,000 miles.

Philosophers whose temperament was inclined to the melancholy, on believing these assumptions, stated that sooner or later a grand crash must occur, and the sun and all the planets be destroyed by a collision with certain fixed stars.

It may tend to reassure those who have heard such statements, to find they have been made by those persons who, however skilful in some branches of science, have yet, when dealing with the proper motion of the fixed stars, omitted to notice that the pole of the ecliptic, which they had assumed to be the centre of the circle traced by the earth's axis, could not be that centre; that the true centre of this circle was not known to these theorists; and that the system adopted to find the proper motion was unsound.

A most interesting fact, however, remains, viz. that when various mathematicians, *accepting as correct the results obtained as above,* proceeded to calculate where

the stars appeared to separate, they all fixed on that point in the heavens which we have already pointed out as the point in the circumference of the true polar circle at which the *rate* in the change of right ascension changes, and which in 1870 had a right ascension of about 265° 4′, and a north polar distance of 58° 53′.

For we find the following as the assigned position of the point towards which it was supposed the solar system was travelling, which point has been technically termed by modern astronomers '*the apex of solar motion*:'

DATE.	RIGHT ASCENSION.	NORTH POLAR DISTANCE.	CALCULATOR.
1790	261° 11′	59° 2′	M. Argelander.
"	261° 22′	62° 24′	Otto Struve.
"	252° 53′	75° 34′	M. Luhndahl.
"	260° 1′	55° 37′	Mr. Galloway.

The point in the circumference of the true circle traced by the pole of the heavens at which the changes would occur has a AR of 265° 4′ and a north polar distance of 58° 53′.

All these results give a point in the heavens so very close to the point in the circumference of the circle actually traced by the pole of the heavens at which the rate of change in right ascension changes, that it is evident this point and the so-called apex of solar motion have an identical cause for being remarkable.

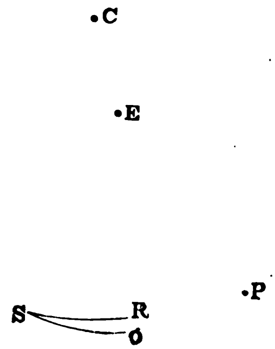
It will be manifest to any person who has followed us in the preceding demonstrations that the arc of

the circle traced by the pole, and over which the stars will not vary their rate of change, will be of some length. Just as the meridian is the point in the course of a celestial body where it changes from rising to decreasing its altitude, and yet scarcely varies this altitude more than a few minutes for four or five degrees east or west of the meridian, so the changes of rate in stars' changes of right ascension, would occur on that part of the polar circle farthest removed from the pole of the ecliptic; yet for a few degrees on either side of that point in the circle the differences would be very slight. Thus we may find for the date 1790 the right ascension of the supposed apex of solar motion given as about 4° less than was that point at which the rates of right ascension changed. Considering the method adopted by former computers to ascertain the position of this point, their close approximation to it is very remarkable.

The true cause of the so-called proper motion of the fixed stars will now be evident to any competent geometrician, also why astronomers have found that a point in or near the constellation of Hercules seemed that point at which the stars appeared to separate. It is not because the whole solar system is rushing in that direction, and thus causing the appearance of the stars separating in that part of the heavens, but it is because the true circle described by the pole of the heavens is at that point farthest removed from the circle which theorists have hitherto supposed the pole traced, and which supposed cir-

cular course has been that on which their calculations relative to the theoretical course of a star have been based.

The difference in the results can be readily understood by a diagram such as the annexed:



Let E be the pole of the ecliptic, P the pole of the heavens, S a star. By the present popular theory relative to the movement of the pole P, the effect is supposed to be the same as if the star S traced part of a circle round E as a centre. (See Article 313, Herschel's *Outlines of Astronomy.*) The course of the star S, on this theory, would therefore be represented by the arc S O round E as a centre. The point E, however, not being the centre, but C being this centre, the apparent course of the star S, on the supposition of representing the polar movement by an actual movement of the stars, would be from S to R round C as a centre.

By the calculation of modern theorists, the star S ought, after a certain period of years, to be located at O, on the assumption that the pole of the ecliptic is the centre of polar motion; when, then, it is found at R, the arc of displacement between theory and fact, viz. O R. is attributed to '*proper motion.*'

Another difficulty, however, has yet to be met. A movement of the pole of the heavens round C as a centre will not necessarily cause E to move, nor will

the star S actually move; consequently the star's colatitude will not change, and E S, the star's colatitude, will be constant; so the effects are not actually the same as though the stars revolved round C, though in some respects the results are similar.

These geometrical results are the cause of the so-called 'proper motion' of the fixed stars.

CHAPTER XI.

GEOMETRICAL LAWS CONNECTED WITH THE CONICAL MOVEMENT OF THE EARTH'S AXIS.

We will now examine certain problems connected with what we may term the transfer of real to apparent motion, and note what the effects are. In some cases these effects would be almost identical; for example, if the whole of the fixed stars revolved round the earth every 24 hours, the results would be the same as if the earth rotated on its axis in 24 hours.

It has been stated by all former writers on astronomy that the effects of the precession of the equinoxes were the same exactly, as if the whole heavens revolved with what we may term right-hand rotation, or from east to west once during 25,848 years, round the poles of the ecliptic.* This statement is incorrect—the effects are not the same, because the pole of the ecliptic is not the centre of the circle traced by the earth's axis.

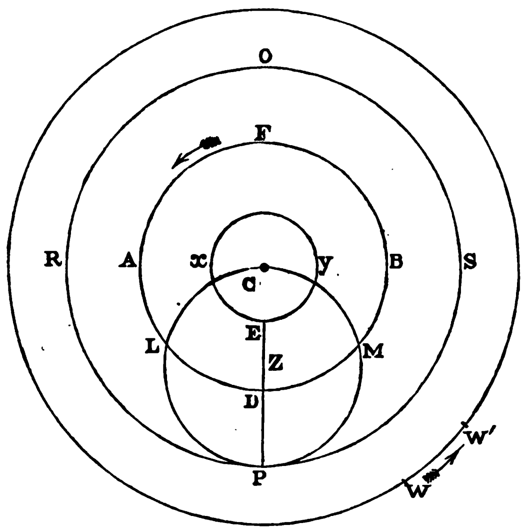
It has also been stated by all previous writers on astronomy that the effect of the precession of the equinoxes was to cause every star in the heavens to increase its longitude uniformly.† This statement is

* See article 313, Herschel's *Outlines of Astronomy*, and all other works on the same subject. † Ibid.

incorrect. The effects would be the same if the pole of the ecliptic were the exact centre of the circle traced by the earth's axis; but as it is not this centre, the effects are different.

We will now consider the difference between the effects supposing we assumed that the heavens rotated round the centre of polar motion, instead of the pole of the heavens tracing its circular course round this point.

Let C represent the centre of polar motion, E the pole of the ecliptic, P the pole of the heavens, all these points being on the surface of the same sphere. Let R P S O, A D B, X E Y, represent three circles on the sphere, the pole C being the centre of these circles.



Let each of these circles represent the apparent course of various stars round C as a centre, on the

supposition that it is the heavens that slowly rotate, and not the earth's axis that changes its direction.

There would now be *three* fixed points on the sphere, viz. C, the pole or centre of the various circles; E, the pole of the ecliptic, which would not move, owing to the heavens rotating; and P, the pole of the heavens. Thus the stars alone would appear to revolve, and we will now consider what these changes would be.

Suppose C X = C E = C Y. The star at X, as it moved towards E, would gradually decrease its distance from E; hence would gradually decrease its colatitude, and thus increase its latitude. A star at A, as it passed down towards D, would also decrease its distance from E by the difference between the length of the arcs A E and D E. A star at B, as it moved round to F, would increase its colatitude by an amount equal to the difference between F E and E B. Now as C E is 6°, the star A would, on reaching D, have 6° (about) more latitude than it had at A. The star B, on reaching F, would have about 6° less latitude than it had at B; and so each star would alter its latitude, but in a somewhat irregular manner. A star on one side of the pole of the ecliptic would not necessarily increase its latitude as much as another star on the opposite side of E decreased its latitude. For example, a star at X would, if this rotation of the heavens occurred, be moving almost directly towards E, the pole of the ecliptic, therefore decreasing its colatitude and increasing its latitude. A star at

W, on the opposite side of E to that on which X is located, and therefore differing 180° in longitude from X, would, by the same movement of the sphere of the heavens that carried X directly towards E, carry W towards W′, which movement would be almost lateral as regards E; a movement which would not cause any very great change in the distance of W, W′ from E, and would not therefore cause any very great change in the latitude of W or W′.

A star in the direction of E W, produced and distant 90° from E, would change its latitude even less than would a star at W, by this apparent rotation of the fixed stars during a long period of time. There would be no variation in the obliquity, owing to this movement; neither E, the pole of the ecliptic, nor P will have been at all affected by the movement of the sphere of the heavens. The stars R, A, X, D, &c. would all have changed their right ascension, and their polar distances from P, much as it is found they do at present.

The radius of the circle, by which circle we determine those points in a star's course where its annual change in right ascension would be zero, would be found by bisecting P C, and finding Z this centre, and not bisecting P E to obtain our centre.

Thus during one half-revolution of the equinoxes many stars would vary their latitude about 12°.

The measure of the rate at which the heavens would revolve can be obtained by the approach of those stars which are moving directly towards P; and

P being distant 29° 25′ 47″ from C, we can obtain the rate of this movement.

The actual occurrences therefore are very nearly identical, though there are marked differences between this movement and that of the pole of the heavens.

We will now point out what these differences are.

First, there would be no change in the obliquity of the ecliptic, and the observations of two thousand years show that a change does occur.

Secondly, every star in the heavens would change its latitude considerably, and in a regular manner; those stars which differed from each other 180° in longitude changing their latitudes almost equally, though in a reverse way as regards + and –.

Thirdly, other celestial bodies, such as the various planets, would not be affected by this movement of the sphere of the heavens; but it is found they are affected by the movement which occurs.

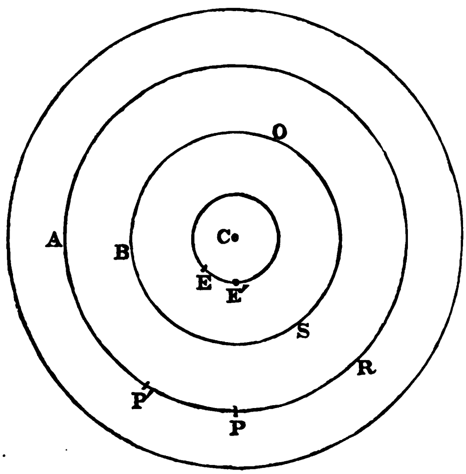
Fourthly, the probability is in favour of this movement being in the earth, instead of its occurring to the thousands of vast suns that are around us.

We will next examine what would be the results of a change in the position of the ecliptic, and in this investigation we are well guided as regards what this change might be, from having already defined the curve traced by the pole of the heavens relative to the pole of the ecliptic.

Referring to diagram, p. 164, we will suppose E the pole of the ecliptic, P the pole of the heavens, P E

therefore the obliquity, C the centre of the circle described by E.

If E moved slowly round to E′, the obliquity would decrease from P E to P E′, *and the effect of the decrease would be almost identically the same as if the pole had moved from* P *to* P′. But if E moved to E′, and thus caused a decrease in the obliquity, this movement of E to E′ would cause the latitude of every star on the meridian of longitude E B A to decrease its latitude by an amount equal to the value of the arc E E′; whilst every star on the meridian E S R 180° in longitude from E B A would increase its latitude a like amount. A star at O would not have altered its latitude, nor would any stars have done so whose longitudes differed 90° from the meridian



E B A. An examination of the changes or supposed changes in latitude of stars, obtained by comparing the earliest catalogues with those framed even now,

yields some evidence of such change in the position of the pole of the ecliptic, and hence of the plane of the ecliptic, during at least 1500 years.

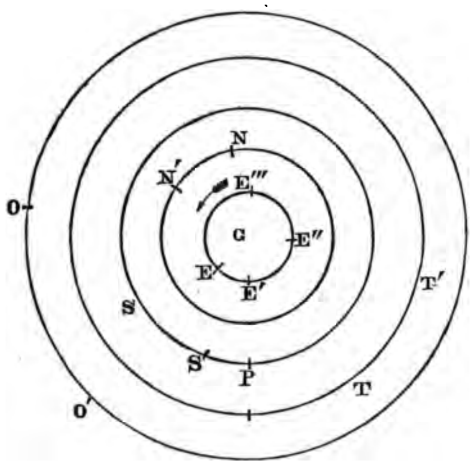
We will now consider the effects of a combination of these two movements, viz. a supposed revolution of the heavens, occurring simultaneously with a revolution of the ecliptic and ecliptic pole round the point C as a centre or pole of the circle.

These movements can at once be transferred as regards their effects to a conical movement of the earth's axis round the point C; for this movement in the earth would produce exactly the same effects on the heavens and on the ecliptic as would the combined movement of the heavens and ecliptic themselves. This is just a similar case to that of the earth rotating on its diurnal axis, which produces effects exactly like those which would occur if each celestial body revolved round the earth every 24 hours.

Referring to diagram, p. 166, let P represent the pole of the heavens, C the centre or pole round which the stars appear to revolve during one revolution of the equinoxes, E the pole of the ecliptic. Assuming the movement to take place on the sphere of the heavens, whilst the pole P is stationary, and that the pole of the ecliptic partakes of this movement, we will first note the effects on E, the pole of the ecliptic.

Let E be the position of the pole of the ecliptic at any date, then P E represents the obliquity. Let E be carried with the sphere of the heavens round C (in the direction indicated by the arrow between

E E‴) to the point marked E′; P E′ will now be the obliquity, and consequently less in value than it was



when the pole of the ecliptic was at E. Continuing this movement of the pole of the ecliptic to E″ and E‴, we should have a continued increase of the distance of the pole of the ecliptic from the pole of the heavens, and consequently a continued increase in the obliquity, until a maximum distance was obtained by the pole of the ecliptic reaching that part of its orbit marked E‴, and which must be a point on the production of the arc joining P and C.

Taking the radius C E as 6°, there would be between the greatest and least angular distance of the pole of the ecliptic from P, the pole of the heavens, a difference of 12°, equal to the diameter of the circle E E″ E‴.

This is just what would occur if the pole P moved

round C, with radius P C, and in the opposite direction to that in which E has moved. We should by such a movement of P, whilst E remained fixed, obtain exactly the same difference of 12° between the least and greatest angular distance of E; and at a first glance the two movements would appear identical in their results. There would be, however, one slight difference, viz. when the arc joining C and E″, or C and E, was at right angles to the arc joining E and P, or E″ and P, then the movement of E would be directly towards or away from P; consequently the angular distance P E would then decrease or increase more rapidly than at any other time for a constant movement in arc of E. If, however, the pole P moved over a given constant arc, it would not decrease its distance from E so rapidly when C E P was 90° as it would when C E P was 135° or 45°. A geometrician will at once perceive why this is.

Taking the sphere of the heavens to revolve with E, the pole of the ecliptic, we should have, for a movement of E to E′, a movement of a star attached to the sphere of the heavens carried from N to N′, a star at O carried to O′, one at T carried to T′, and so on. A star at S would be carried to S′, and the rate at which this star approached P when P S was at right angles to P E would be the only guide by which an observer could judge of the rate at which this apparent rotation of the heavens was being carried on.

If he found that the star at S approached P at the rate of 20″ annually, and the only guide he had was

this 20″, and the consequent variation of the angle S′ E P, he would probably, if he omitted to notice the decrease in the angular distance of P E, conclude that the rate of an entire revolution of the equinoxes was to be measured by this rate of 20″ annually, counted on a circle of the sphere just as distant from the pole of the sphere as P was from E. If, then, P E was 23° 28′, the circle on which P was situated would contain 360° × sin. 23° 28′ = 143° 21′. If, then, 1′ occupied three years, 143° 21′ would occupy 25,803 years; *and this is the erroneous method which has hitherto been practised.*

Such a calculation, however, would only be made by those who failed to note that E, in consequence of decreasing its distance from P, could not be the centre of this interesting apparent motion.

What the geometrician has to deal with in order to find the rate at which this apparent rotation of the heavens is occurring is the angular distance P C, not the variable distances P E, P E′, &c. Now P C is 29° 25′ 47″, and when we find that on a circle removed from C, this amount, an arc appears to be traced annually of 20″, or 1′ in three years, we must not make use of a point at a variable distance from P, like E, the pole of the ecliptic, but a constant point like C, the centre of this circle. We may then calculate how many degrees this circle contains as seen from the centre of the sphere, and making use of the same formula as in the last case, but substituting sin. 29° 25′ 47″ for sin. 23° 28′, we obtain 176° 52′ for

the degrees in this circle. Then, at the rate of 1′ in three years, we obtain 31,836 years for the period of an entire revolution, *and this appears to be the true period of a revolution of the equinoxes.*

It will be seen that the movement of every star in the heavens would be such as to cause it to produce a change in its right ascension or declination from time to time, but this movement would be slightly different from that which would occur, if the various stars appeared to move in orbits round E as a centre.

The movement of E, the pole of the ecliptic, with the whole sphere of the heavens, as if the two were part of the same rigid body, will cause E to maintain a constant distance from every star; thus, as the stars shown on diagram, p. 166, move round (or rather *appear* to move round, for we must not forget that this movement is an appearance only, produced by a corresponding movement in the earth), their distances from E never vary; thus E N = E′ N′, E T = E′ T′, and so on. Consequently there would be no *actual* change of latitude in any star by this movement.

We can now transfer the actual motion to the earth itself, just as we can the effects of a diurnal rotation, and we can perceive what are the effects of this motion round an axis directed to a point located as is C. But the instant we know that this apparent motion of the sphere of the heavens and of the pole of the ecliptic is due to a sort of second rotation of the earth itself, we have another important fact to consider, viz. that no amount of rotation of the earth

or change in direction of its axis will, *per se*, cause the slightest variation of the *course of the earth round the sun*. Thus the ecliptic, as traced out at one time by the sun's apparent course, will be the same in position, no matter what variations occur in the earth's axis. But we have seen that this change in direction of the axis and the conical movement round the point C is exactly represented by supposing that the sphere of the heavens rotates round this point. Thus if a star were 90° distant from C, it would appear to be carried round a great circle of a sphere of which C was the pole, and thus it would not move exactly parallel to a circle which had the point E for its pole. But as this effect would still be due to the movement of the earth, it would be an *appearance* only, and not an actual fact.

Enough, however, has been demonstrated, we believe, in the preceding pages, to show that we cannot with truth state that a star has what is called 'a proper motion' until we have correctly localised the centre of polar motion, and thus defined the exact radius of the circle described by the pole of the heavens; and this point and this radius has never been accurately defined until it was announced in our paper read before the Royal Astronomical Society in 1870, and the fact demonstrated in our work, *The Date, &c. of the Glacial Epoch.*

We have also pointed out that there is an error in principle in attributing to a star 'a proper motion,' because the mean of the extreme rates of change of a

star, multiplied by the number of years between the dates at which these rates are given, is found not to agree with the star's actual position, when this result is added to or subtracted from the star's position at either date.

Thus, before we can announce that any star has a proper motion, we must not only adopt a different system for attempting to ascertain this proper motion, but we must be certain that some unknown movement does not occur in our earth, and which will produce apparent changes in the positions of the various celestial bodies. The old practice of comparing right ascensions at different dates, and drawing conclusions from the differences then found, will merely lead to confusion. To state that a star has a motion in a given direction when we do not know whether this star is within or without the circle described by the pole, and do not know its angular distance from the centre of polar motion, is as premature as if a surveyor announced that church spires, chimneys, trees, and flagstaves were moving about, and had 'a proper motion,' because during his observations his theodolite made one movement when he supposed it made another.

The earth with its transit is, to all intents and purposes, our theodolite. It has been supposed that this vast instrument has a conical movement of its vertical axis round another axis inclined to its vernier plates at an angle of 66° 32′; whereas it has a movement round an axis inclined to these plates at

an angle of 60° 34′ 13″ only. Consequently the observations made with the instrument are different from those which it is supposed ought to result.

Thus, that the fixed stars have a proper motion is not as yet proved, in spite of all that has been written and said on the subject during the past hundred years.

If the pole of the ecliptic were the centre of polar motion, the effect would be the same as though the heavens themselves rotated round this pole. Consequently every star in the heavens would, during a long period of years, appear to rotate round the pole of the ecliptic, just as the various stars do round the pole of the heavens every 24 hours. Every star would change its longitude by an equal amount, and every star would retain the same latitude. The obliquity of the ecliptic would be a constant quantity, and we should have no difficulty in determining stars' positions for any date in the future.

The conversion of longitudes and latitudes of stars into right ascensions and declinations is readily accomplished by the formula already given. Consequently from one catalogue we could at once frame another for any future date, because we should know that the longitude of every star changed uniformly, and the latitude remained constant.

If, however, the plane of the ecliptic shifted its position, there would be a change in the latitude of every star owing to this change; but if we knew what was the amount and direction of this change

(as we ought to easily discover, if it occurred, by the manner in which stars' latitudes changed), we could easily calculate its effects, allow for these, and still frame our catalogue of stars for the future.

If, however, the plane of the ecliptic changed its position among the fixed stars, and the pole of the heavens still continued tracing a circle round this pole as a centre, both right ascensions and declinations of stars would be altered in almost exactly the same manner as latitudes and longitudes of stars, for just as much as the pole of the ecliptic moved towards or away from any stars, just so much would the pole of the heavens move towards or away from the same stars; so that this effect also could be calculated, and a star catalogue for the future easily framed on sound principles.

If the plane of the ecliptic changed its position, and the pole of the ecliptic still remained the centre of polar motion, there would be no variation in the obliquity of the ecliptic; consequently, when we find that there is a variation, we know that the above conditions are not fulfilled.

Next, if the pole of the ecliptic were at any one instant the centre of polar motion, and then by a slow movement of the ecliptic it ceased to be the centre of polar motion, whilst the pole of the heavens still traced its circular course round that same centre, there would be no discordances in the right ascensions or declinations of stars, if we referred the right ascensions to that point in the heavens which was

and had been the centre of polar motion. If, however, we referred our right ascensions in any way to the pole of the ecliptic, and called the pole of the ecliptic a zero of 18 hours' right ascension, we should at once admit confusion into our catalogues, because, unless we knew exactly the amount and direction of the motion of the pole of the ecliptic, we could not tell in what way our right ascensions should vary.

Again, if we did not know where our centre of polar motion was located, we could not tell how the angle between this centre and the pole of the ecliptic varied, in consequence of the movement of the pole of the heavens. Thus we might be lessening the angle between the two poles, or increasing the angle between them, and therefore increasing the frequency of its successive transits, or decreasing them, as the case might be. Consequently we have to consider the effects resulting from the fact of a decrease in the obliquity now known to occur; and in order to examine this problem thoroughly, we are unavoidably led to examine another intimately connected with and affecting the supposed proper motion of the fixed stars. This problem is the various methods of measuring time, and the means adopted for obtaining a standard measure of time. Before it can be stated that the stars have a proper motion, this problem of time must be thoroughly understood, for it is connected with changes in the right ascension of celestial bodies. We will therefore shortly quit the subject

of the proper motion of the fixed stars, and turn our attention to 'time.' We must first, however, examine another subject connected with the conical movement of the earth's axis.

CHAPTER XII.

MOVEMENTS OF THE EARTH WHICH MAY ACCOMPANY THE CONICAL MOVEMENT OF THE EARTH'S AXIS.

When the astronomers of the two past centuries had discovered that if they attributed to the axis of the earth a conical movement round the pole of the ecliptic as a centre, they explained the bare fact of *a* precession of the equinoctial point, and gave a geometrical cause for the precession, they believed they had exhausted this problem, and had solved every item of it. When, in addition, Sir Isaac Newton explained this conical movement of the earth's axis by attributing it to the action of the sun and moon on the protuberant equator of the earth, superficial theorists believed that the theory connected with the change in direction of the earth's axis was thoroughly worked out, and they supposed that, no matter what branches of astronomy might offer a fruitful field for research, the problem connected with the conical movement of the earth's axis was one in which nothing more remained to be ascertained.

Two grave oversights were committed in connection with the former theories relative to this subject.

The first was supposing that the pole of the ecliptic was the centre of the polar circle, and the second was supposing that the whole period of a revolution of the equinoxes might be arrived at by multiplying the time occupied in advancing 1° by 360.

At the time when it was supposed that the earth's axis traced a circle round the pole of the ecliptic *as a centre*, the astronomers were not aware that there was a secular decrease in the obliquity of the ecliptic. Sir Isaac Newton shows by his writings that he was as unacquainted with the fact of a decrease in the obliquity as he was with the existence of the planets Neptune and Uranus, and of the hundred and odd asteroids between Mars and Jupiter. Had he known of such decrease, he would very soon have modified some of the theories based on the assumption that the pole of the ecliptic was the centre of polar motion.

We find, then, that the assumption of the pole of the ecliptic being the centre of polar motion, and of the pole of the ecliptic being always the centre of the circle traced by the earth's axis in the heavens, is not correct, and that when we trace out the curve along which the pole does move, it proves to be a circle the centre of which is 6° from the pole of the ecliptic. The results which follow this movement are very different from those which would occur if the pole of the heavens traced a circle round the pole of the ecliptic as a centre. The changes in the various right ascensions of every star would be different in

each case, certain changes occurring with one movement which would not occur with the other. Some stars which changed their right ascensions in strict geometrical accordance with the one movement, would exhibit changes due entirely to this movement, which would be totally inexplicable if the pole of the heavens traced a circle round the pole of the ecliptic as a centre. It is not difficult therefore to understand that, when it was omitted to be noticed that the pole of the heavens moved in a manner impossible according to old theories, even modern speculators could find no explanation so ready, and apparently so easy to escape from a difficulty, as to attribute to nearly every star a proper motion of its own.

Just as the ancients set to work to invent another epicycle when those already adopted failed to explain facts, instead of reëxamining their then orthodox beliefs relative to their astronomy, so their more modern imitators have invented theory after theory to explain those discordances due entirely to the idea that the earth's axis traced a circle round the pole of the ecliptic as a centre. One of these theories is, that the discordances in stars' right ascension, unexplained by the theory of the pole of the ecliptic being the centre of polar motion, are due to the movement of the stars themselves, and not to any incompleteness in the knowledge of what such movement of the pole really is.

We have already pointed out the recorded facts

known relative to the decrease in the obliquity of the ecliptic, and have demonstrated that this decrease gives us the true curve traced by the earth's axis on the sphere of the heavens. When we have defined this curve we have solved only one portion of that problem, every part of which it is necessary to solve before we can define what will be the actual changes in right ascension produced by that movement of the earth hitherto supposed to be fully defined when it was stated to be a conical movement of the earth's axis round the pole of the ecliptic as a centre.

To know that the earth's axis traces a circle in the heavens during a given period, and to know that it changes its direction annually to the amount of about 20″, are two useful pieces of information; but until we know something more relative to the course of the pole, and also relative to the movement of a meridian as affected by this conical movement, we cannot do more than guess at the effects which result therefrom.

To state that the axis of a sphere traces a cone is a very incomplete definition of what the sphere itself does, for the sphere may be influenced in several different ways during the time that the axis is describing its cone.

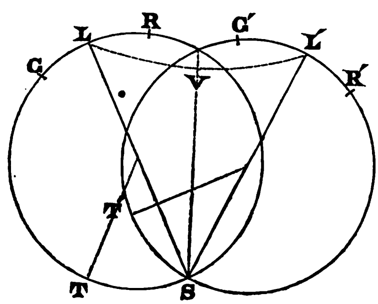
In order to point out some of the varied results which may follow or occur with a conical movement of an axis of a sphere, we will treat of this conical movement only, without mixing it up with a diurnal

rotation; but it will be evident that whatever be the movement of the sphere depending on or affected by the conical movement of the axis, this motion will affect the results of a diurnal rotation, and in long periods of time will be plainly manifested.

If, then, the earth consisted of an axis only, the definition 'a conical movement of this axis' would probably be sufficiently explicit to give all requisite information; but when we are investigating such a problem as 'time' and its minute divisions, we must ascertain *what happens to a meridian* as well as to an axis.

We will now refer to four different cases connected with the conical movement of the axis of a sphere, and endeavour to explain the results which would occur in each case.

Problem 1. Let G′ L′ R′ S represent a sphere, of



which the axis is L′ S. Let R′ and G′ be two points on the surface of this sphere.

Let the axis L′ S trace the half of a right cone; viz. let S remain at rest whilst L moves over the semicircle L′ V L. Let the angle L S L′ = 58° 51′.

Let the arc G′ L′ R′ be directed towards an infinitely distant line θ, and let the same arc, G L R, be directed towards the same distant line θ when the axis is in the position L S.

By such a movement the axis L′ S has traced out half a cone, the angle at the vertex of which cone is 58° 51′. In like manner, the points R′ and G′ have each traced out the half of a cone, the angle at the vertex of which would be in each case R′ S R and G′ S G, which would each be equal to L′ S L = 58° 51′.

Let T′ be another point on the sphere G′ L′ R′ S. The point T′, under the above conditions, would occupy the position T after the axis L′ S had traced half its conical movement, and the point T′ would have traced out half a cone, the vertex of which was S′, and the angle at the vertex T′ S T.

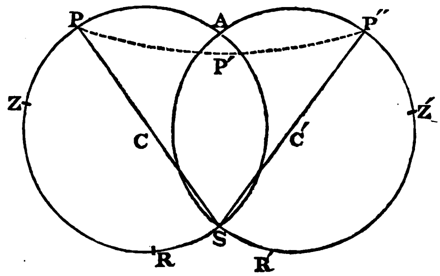
By giving to a sphere or planet a conical movement of the axis similar to the above, each zenith on that sphere traces out a cone during the entire conical movement of the axis. The value of the angle at the vertex of the cone being in all cases equal to the angle at the vertex of the cone described by the axis.

The direction in which these cones point is limited, and is never in the opposite direction to that in which the vertex points of that cone formed by the axis.

This conical movement of the axis of the sphere will be called No. 1 *in future references;* and the reader will find the after investigations simplified if he take

a globe, or merely a sphere, and examine practically the effects of this movement of the axis.

Problem 2. Let A P″ Z′ R′ S be a sphere, of which



the axis is P″ S. Let Z′ and A be two points on the surface, as also R′, and let the great circle passing through A P″ and S also pass through Z′ and R′.

Let the axis P″ S trace the half of a cone; viz. let S remain at rest, whilst the point P″ traces the semicircle P″ P′ P. Let the angle P″ S P be 58° 51′.

Let the point Z′ move round with P″, so that Z′ occupies the position Z, and R′ occupies the position R, when half the conical movement has been traced out by the axis P″ S.

Suppose the arc P″ Z′=39°. The zenith of Z′, upon reaching Z, will have traced half a cone, the angle at the vertex of which will be Z′ S Z, which is equal to 58° 51′+Z′ S P″+Z S P, which amounts to 136° 51′.

The point R has traced half a cone, the angle at the vertex of which is R S R′, and if we take the arc S R′=30°, the angle R S R′ will amount to 163°.

During this half conical movement, the points S and A have remained at rest, and thus the point Z′ may be said to have traced a half circle around that point in the heavens towards which S A is directed.

In consequence of the two points S and A remaining at rest whilst the axis P″ S traces half the cone P′ S P, this conical movement of the axis may be transformed, as far as its geometrical effects are concerned, into a rotation around a second axis S A. For by keeping A and S fixed, and giving to the sphere A P″ Z′ R′ a semi-rotation round the axis joining A and S, we find P″ moves along the arc P″ P′ P, and Z′ moves to Z, and R′ to R, just in the manner already mentioned and shown in the diagram.

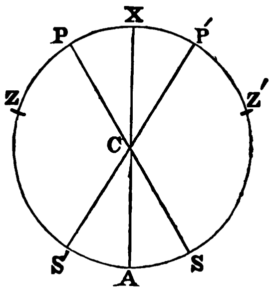
This conical movement of the axis of the sphere will be called No. 2 in future references.

* * * * *

In both No. 1 and No. 2 it will be observed that the axis itself traces exactly the same course. It traces during a complete revolution a cone, the apex of which is S.

The pole P″ or L′ in each case traces a circle, during a complete revolution, around a fixed point in the heavens, that fixed point being distant 29° 25′ 30″ from the direction of the axis S P″, S L′, &c. The marked differences in the two cases being the different directions in which various zeniths point after a certain amount of the conical movement of the axis has been performed.

Problem 3. Let P′ Z′ S S′ Z represent the earth; the axis of the earth being P′ S′.



Let the line A X pass through the earth's centre, and be directed to the pole of the ecliptic, and let Z′ be a point on the earth's surface on the meridian P′ Z′ S.

Let the pole P′ trace half a circle round the pole X of the ecliptic, whilst C, the centre of the earth, X and A, points on the surface, are at rest. The pole being thus moved round a half circle, the semi-axis of the earth P′ C would trace a half cone P′ C P, and the remainder of the axis S′ C would also trace a half cone, viz. S′ C S, in the same period.

The zenith which was at Z′ would have been carried round to Z by this conical movement of the semi-axis, and the following effects would have occurred:

The pole of the heavens would have described a semi-circle around the pole of the ecliptic, maintaining from it a uniform distance.

There would have been a precession of the equinoxes during the whole of this period, at the rate of 50″ annually to a rate of 20″ in the angle P′ C P.

Every star in the heavens would have increased its longitude uniformly.

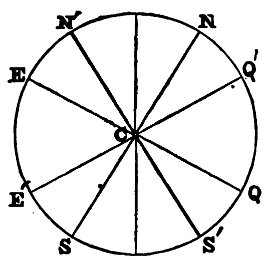
The visible appearance would have been as though

the heavens had made a slow but complete revolution, from west to east, round the poles of the ecliptic.

In fact, by the conical movement of the semi-axis, No. 3, the *principal* effects would be identical with those which would occur by the conical movement of the whole axis, No. 2.

To note what differences would occur in the details is the work of the geometrician.

Problem 4. Suppose the semi-axis N S of the sphere N Q S E to trace half a cone during a given interval of time, and at the end of this interval suppose the axis to occupy the position N′ C S′. This change in direction of the *axis* is exactly similar to that demonstrated in Problem 3.



Suppose, however, that points Q and E on the equator become, by this change in direction of the axis, unchanged as regards their relative direction towards a fixed celestial meridian, they would, under such conditions, occupy the positions Q′ and E′ when the axis was at N′ S′.

This change in direction of the semi-axis, assuming the angle N′ C N to be as before, would cause the precession of the equinoxes nearly in the same manner as those changes mentioned under the heads of Problems 1, 2, and 3.

Hence there are four utterly different conical

movements of a planet's axis or semi-axis, all of which will explain:

The precession of the equinoxes,

The difference in the length of the solar and siderial year,

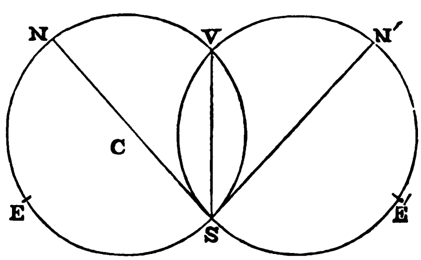
The uniform changes in stars' longitude,

And certain variable changes in right ascensions and declinations.

Summary of the Four Movements.

Problem 1 is a conical movement of the earth's axis, which causes this axis to trace a cone in a given number of years. In this conical movement no part of the earth's surface remains at rest during the period, and referred to this movement only, except the south pole.

Problem 2 is a conical movement of the earth's axis, which causes this axis to trace a cone in a given number of years, and the axis itself traces exactly the same course that it did by Problem 1. In this conical movement two points on the earth's surface remain at rest during the period, viz. the south pole



and also a point distant 58° 51′ from the north pole

of the earth, as may be shown in the annexed diagram. Let N S be the first position of the axis, N′ S its position after tracing the half cone N S N′. Let E by this movement be carried to E′. By driving an axis through the sphere N E S in the direction of V S, and causing the sphere to form half a rotation around this axis, the first axis N S is moved to N′ S, and E to E′.

If the axis N S has traced a circle round a fixed point in the heavens towards which S V is directed, maintaining a uniform distance of 29° 25′ 30″ from this point, equal to the angle N S V, we then have the following:

The angle N S V=29° 25′ 30″, equal to C V S, because C V and C S are radii. Then the exterior angle N C V=C V S+C S V, equal therefore to 58° 51′.

This conical movement of the axis is the same thing as a rotation from east to west around a second axis, which second axis passes through the south pole and a point on the earth's surface distant 58° 51′ from the north pole.

This axis might vary in position almost indefinitely; thus it might pass through the north pole and a point 58° 51′ from the south pole, whilst its longitude might be a constant or a variable quantity. But as long as we state that the earth's axis traces a cone, either the north or south pole of the earth must be assumed to be a fixed point as regards this conical movement, and we naturally inquire *which pole is assumed fixed, and upon what evidence was it so assumed.*

Problem 3. Whilst Problem 2 may be described as a slow rotation of the earth from east to west around an axis which passes through one pole and a point 58° 51′ from the other pole, thus giving to the axis a conical movement, Problem 3 may be described as a slow rotation from east to west around an axis which passes through the earth's centre, and through two points, one distant about 23½° from the north pole, the other 23½° from the south pole.

Problem 4. This problem is a slow conical movement of the semi-axis of the earth instead of the whole axis, and differs from Problem 1 in this respect; no rotation of the earth occurring with the conical movement.

CHAPTER XIII.

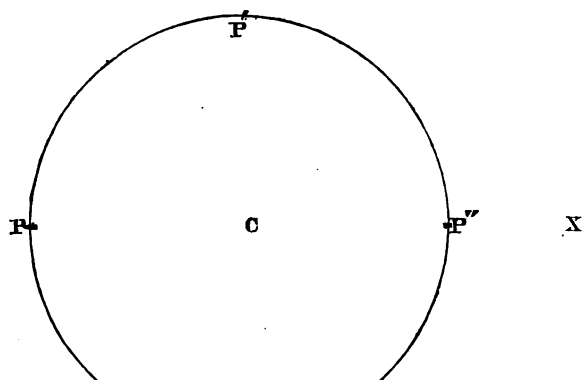
VARIED RESULTS ATTENDING THE SAME MOVEMENT OF THE EARTH'S AXIS.

It will be seen from the four problems in the last chapter, that to define a movement of the earth as a conical movement of the axis is a very imperfect definition. There are two exactly similar conical movements of the axis which *can* occur, and yet the combining movements of the earth are in the two cases very different. By the one movement, an actual rotation of the earth occurs, and in fact produces as a geometrical result the conical movement of the axis. By the other movement, no rotation of the earth is combined with the conical motion.

During an entire revolution of the equinoxes there would be one more solar day by the one movement than by the other, also each meridian would be very differently affected by the one movement than by the other; consequently the various right ascension of stars would be changed according as the various meridians were differently affected; therefore the details of this problem become interesting subjects for examination from a purely geometrical point of view.

Let N represent the number of rotations of the earth on its axis during an entire revolution of the equinoxes, and consequently during the time occupied by the pole of the heavens in tracing its circle round the pole of the ecliptic. If the movement of the earth were similar to that described in Problem 3—that is, a conical movement of the axis, not combined with one rotation during that whole period—then a star within the circle described by the pole would pass the meridian once oftener than would a star outside that circle; a fact which may be demonstrated and made more intelligible by aid of the following diagram.

Let C be the centre of polar motion, P the position of the pole at a given date, X the direction of a



star outside the polar circle. Let a star be at C within the circle; a star at C would transit simultaneously with the star at X when the pole was at P. When,

however, the pole was at P$''$ there would be an interval of twelve hours between the transit of C and that of X, and C would have gained on X and be twelve hours in advance of it. On the pole completing its circle and reaching P again, C would have transited once oftener than would X during this long interval of time.

This same problem may likewise be understood if we call attention to the fact, that if this conical movement of the axis occurred, *and no diurnal rotation of the earth on its axis*, the same side of the earth would always be turned towards the star X, but not always towards the star at C.

If the conical movement of the axis of the earth occurred with that rotation of the earth which might produce the conical motion, then, as the pole slowly moved round the centre C, the same side of the earth would always be towards C, and therefore opposite sides of the earth would be turned towards a star situated as is the star at X, and to all stars situated outside the circle traced by the pole. Therefore, by this movement of the earth combined with the conical movement of the axis, there would be one more transit of a star outside the polar circle during a revolution of the equinoxes than there would be of one inside the circle.

The effects of this movement during a whole revolution of the equinoxes would be the same as if an axis were fixed in the earth and remained pointed towards the centre of polar motion during the whole

period, that is, omitting notice of the diurnal rotation and treating of this motion alone.

Thus, by exactly the same movement of the earth's axis, it is possible that two entirely different movements of the earth itself may occur. By one movement, a star outside the polar circle does not transit each meridian on the earth, whilst a star within the circle does do so. By the other movement, a star without the polar circle does transit every meridian on earth, but a star within the same circle does not do so.

When we examine this same problem as regards the transits of the sun during an entire revolution of the pole of the heavens round the centre of polar motion, we also find most singular variations, according as one or the other movement occurs in connection with that of the axis. If the movement of the earth were similar to that mentioned in Problem 3, and which was a conical movement of the axis not produced by a rotation of the earth, the results as regards transits of the sun may be understood from the following diagramsand explanations.

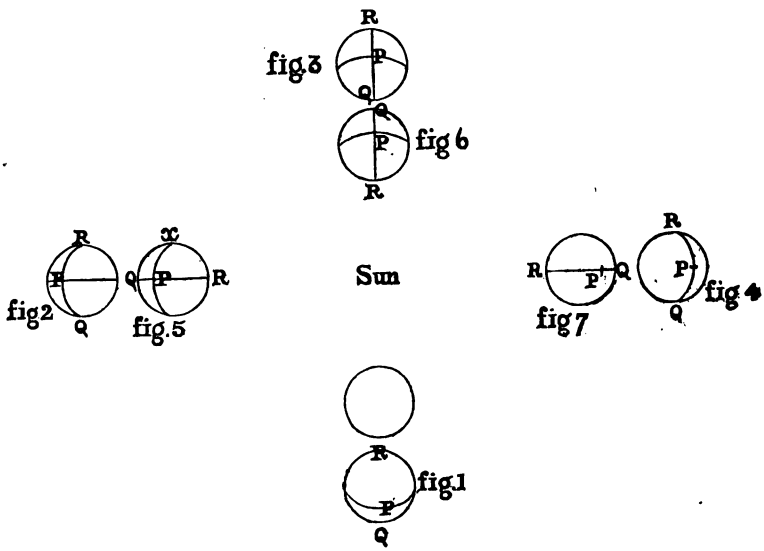
Fig. 2 represents a plan of the earth's orbit projected on the plane of the ecliptic. P being the north pole and P R Q a given meridian.

The problem for investigation now is, in what manner does the meridian Q P R, fig. 2, change in consequence of and due entirely to that movement which produces the conical movement of the axis.

The conical movement of the axis being from left

to right, we show in the following diagram the two results which would occur to a meridian, according as one or other of the movements takes place.

In the diagram, fig. 1 represents the position of



the earth at a given time, P the earth's pole, R P Q a given terrestrial meridian. If the earth's axis traced its conical motion without the combining rotation, the position of the meridian R P Q, fig. 1, would be R P Q, fig. 2, when one-fourth of the circle had been traced by the pole P round the centre of polar motion; that is, the meridian, *owing to this movement alone*, would occupy the position shown by R P Q, fig. 2. And any change in this meridian would be due to the diurnal rotation of the earth on its axis.

If, however, the conical movement of the earth's axis be combined with and due to the slow rotation

of the earth from left to right, then on the pole P reaching exactly the same position as in the last case, and *the axis* having moved exactly in the same manner as in the last case, the meridian referred to would occupy the position Q P R, fig. 5, instead of the position shown by Q P R, fig. 2.

Now as the diurnal rotation of the earth is from right to left, it follows that for the meridian Q P R, fig. 5, to occupy the position shown by Q P R, fig. 2, one-fourth of a diurnal rotation of the earth must take place; consequently during *the interval of standard time* occupied by the earth's axis and pole in reaching the position shown by P, figs. 2 and 5. Six hours more of mean solar time would have transpired if the movement were that indicated by fig. 2 than would have transpired if the movement were that indicated by fig. 5. In other words, it would be six o'clock P.M. on the meridian P R if the movement were that shown by diagram, fig. 2, and noon on the same meridian if the movement be that shown at fig. 5.

If we represent the number of solar days that would occur during the movement of the pole from the position shown by fig. 1 to that shown by fig. 2 by N, then we should have the following equation, viz. $N = N' + \frac{1}{4}$ of a solar day, where N' indicates the number of transits of the sun over the meridian P R, according to the movement shown by fig. 5.

Carrying on this demonstration, we find that at fig. 3 R P Q represents the position of the given meridian according to movement Prob. 1, Q P R, fig. 6, the

position of the same meridian according to the movement Prob. 2. At this point it is evident that the meridian P Q R is by Prob. 2 reversed, and consequently there would have occurred by movement Prob. 1, shown at fig. 3, half a diurnal rotation more than there would have occurred by the movement described as Prob. 2, and shown by the position of the meridian Q P R, fig. 6.

It must be borne in mind that in both cases, viz. that shown by fig. 3 and that shown by fig. 6, the earth's axis would occupy exactly the same positions. The pole would be in exactly the same position both as regards space and as regards its position on the sphere of the heavens, and the earth's centre would be in exactly the same position as regards the sun, the fixed stars, and the ecliptic. So that if the earth's centre, a star on the ecliptic, and the sun's centre were in the same straight line in one case, they would be in the same straight line in the other case, the only differences being that connected with the conical movement of the earth's axis and the resulting movement of the earth itself; differences, however, which will cause most marked changes in the measurements of time and in the variations of stars' right ascensions.

On the earth's axis having performed three-fourths of its conical movement, the pole P will occupy the position shown by P, fig. 4, and the meridian R P Q as there shown will be the position of the meridian indicated by R P Q, fig. 4. If, however, the

movement of the axis be combined with the movement described by Problem 2, then the meridian R P Q, fig. 1, will occupy the position shown by Q P R, fig. 7; and to move into the position shown by R P Q, fig. 4, three-fourths of a diurnal rotation of the earth, equal to 18 hours, must occur.

On the axis completing its conical movement, and the pole completing its circle round the centre of polar motion, the pole would again occupy the position shown by P, fig. 1, and the meridian R P Q would again occupy the position shown by R P Q, fig. 1. *And this would be the position of the meridian after a complete conical movement of the pole, no matter which movement of the earth accompanied the conical motion of the earth's axis.*

The four positions of the earth shown by figs. 1, 2, 3, 4 represent the positions of the earth at the period of the winter solstice; so that the completion of this circuit represents a complete revolution of the winter round the ecliptic, which is the same thing as the revolution of the equinoxes; hence that which is here represented is that which may occur during one revolution of the equinoxes, that is, during about 31,500 years.

We may now abstract the main facts as regards these two movements, and it will be found that, according to the movement shown by figs. 1, 2, 3, 4, there would be, owing to the movement of the axis, one more transit of the sun than there would be by the movement shown by figs. 5, 6, and 7. There

would be also one more transit of a distant star than there would be if the movement were that shown by figs. 5, 6, and 7.

The results affecting stars within and without the circle traced by the pole would be singular. First, we will refer to that which would occur if the movement were that shown by figs. 2, 3, and 4. By this movement a star considerably removed from the circle traced by the pole, and say on the meridian Q P R, and vertical at R, fig. 1, would be on the same meridian very nearly at figs. 2, 3, and 4, and would not consequently transit a meridian even once during an entire revolution of the equinox, unless a diurnal rotation occurred. A star, however, within the circle traced by the pole, say near the pole of the ecliptic, would be passed by the meridian Q P R once, and would return to this meridian during one revolution of the equinoxes, and the effects would be the same as if this star slowly rotated round the centre of polar motion; therefore there would be one more siderial day for a star within the polar circle than there would be for a star without this circle by the movement shown by figs. 1, 2, and 3.

By the movement shown by figs. 5, 6, and 7, a star without the circle traced by the pole, say a star on the meridian Q P R, fig. 1, and vertical at R, would be on a meridian at right angles to P Q R, viz. such a meridian as P X, fig. 5, when the axis had traced one-fourth of its conical movement. For the star X to come on to the meridian P R, one-fourth of a *diurnal*

rotation must occur. Consequently the effect would be the same as if the star had moved round 90° from right to left. On the earth reaching fig. 6, the star which was on the meridian P R, and above the pole, would be on the meridian P Q, or below the pole, as regards the meridian P R; that is, the star has lost 180°, or 12 hours, as regards this meridian. Carrying on the same demonstration, it will be evident that a star outside the circle traced by the pole transits a meridian once from right to left, or in an opposite direction to that of the diurnal rotation, during one entire revolution of the equinoxes by the movement shown by diagrams 5, 6, and 7.

There would thus be one siderial day more during a revolution of the equinoxes by the movement shown by figs. 2, 3, and 4 than there would be by the movement shown by figs. 5, 6, and 7; for a star, by the movement shown by 5, 6, and 7, would appear to move round from right to left, or in opposition to the direction of diurnal rotation, once during a complete revolution of the equinoxes.

A star situated exactly at the centre of polar motion would, as far as the conical movement of the axis was concerned, remain always on the same meridian, if the movement of the earth producing the conical movement were that described as Prob. 2; but this star would appear to make one rotation from east to west during one revolution of the equinoxes if the movement were that described as No. 1.

The combinations of movements connected with these two rotations are numerous, and produce varied results on the relative measures of the solar and siderial day, and also on the changes in a given meridian, and hence in the changes of right ascension of a star. Our object in referring at some length to these changes, and the importance of an investigation of them, will be spoken of in the next chapter.

CHAPTER XIV.

VARIOUS MEASURES OF TIME.

IT is customary to speak of a day as the interval of time occupied by the earth in rotating on its axis. Those persons who have directed their attention to this subject, however, will have remarked that three kinds of days are referred to in those works which treat of astronomy.

First, reference is made to a siderial day.

Second, to a solar day.

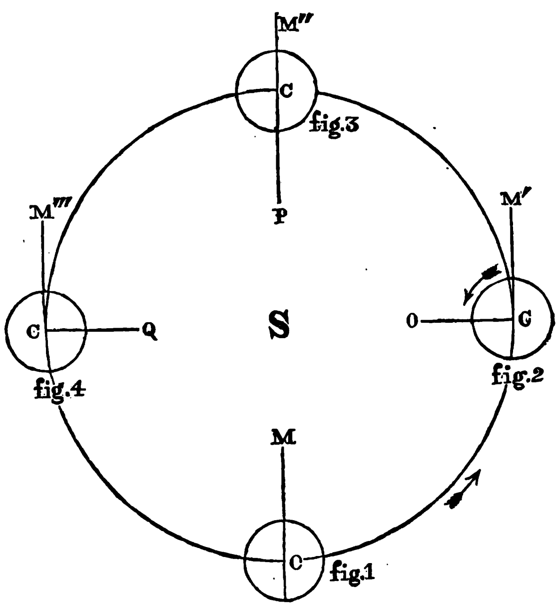
Third, to a mean solar day.

From the use of these three definitions it is evident that we cannot define *a* day as the time occupied by the earth in rotating on its axis, unless we state to which day we refer. Thus we will, in the first place, show why there should be a difference between a siderial and a solar day, and then proceed more into details.

In commencing this inquiry we will take as granted that the earth does rotate on its axis, and, as a preliminary step, that this rotation is uniform. Secondly, that the earth revolves round the sun, and traverses 360° in about 365¼ days. In these pre-

liminary demonstrations it is not our intention to enter on minutely accurate details, but to give only such data as is requisite in whole numbers. An explanation given by such means is usually more readily understood than if the mind is occupied and burdened with minute fractions, which are unnecessary for demonstrations, although essential when we come to details. Thus when we speak of the earth passing over 360° of its orbit in 365¼ days, we do not mean that this is the exact rate at which the earth travels, but it is sufficiently near to enable us to demonstrate the problem which immediately follows.

On the annexed diagram, fig. 1 is intended to



represent a plan of the earth at that date when the

sun S, and a star at an infinite distance, are on the meridian at the same instant. C M we take as a projection of the meridian which passes through S and the distant star. Three months after the earth was at fig. 1 it would have passed over 90° of the ecliptic, and would have reached the position indicated by fig. 2.

When at fig. 2, the star and the sun would not now be on the meridian at the same instant, for the meridian C M′, which is directed to the distant star, would have to rotate to the position C O before it came to the sun. Now as the angle M′C O is 90°, or one-fourth of 360°, so also this angle, represented in time, is 6 hours, or one-fourth of 24. Thus the star has gained 6 hours on the sun during the passage of the earth from fig. 1 to fig. 2.

When the earth has reached the position shown by fig. 3, the meridian C M″ would, after being directed to the distant star, have to rotate 12 hours before it occupied the position C P, and was directed towards the sun. Thus the sun would have lost 12 hours as regards the star during its journey from fig. 1 to fig. 3.

When the earth reached fig. 4, the meridian C M″ would have to rotate round 270°, or 18 hours, to occupy the position C Q, and bring the sun on this meridian. Thus the star has gained 18 hours on the sun during the passage of the earth from fig. 1 to fig. 4.

On the earth reaching the position fig. 1 a second time, and bringing the sun and the star again simul-

taneously on the meridian, the star has gained 24 hours on the sun, and has passed the meridian once oftener than the sun.

Now it is of no matter as regards the truth of this demonstration whether the earth rotate 20 times or 365 times during its revolution round the sun, but whatever number of times the sun is brought on the meridian during this revolution, the star will be brought on once oftener.

We define a solar day as the interval of time between two successive transits of the sun over the same meridian, and a siderial day as the interval between two successive transits of a star over the same meridian, and we find that during one year there are about 365¼ transits of the sun and 366¼ transits of a star.

In order, then, to find the value of a siderial day in terms of a solar day, we have a simple proportion, for a solar day of 24 hours will be to a siderial day as 366¼ is to 365¼. Upon working-out this proportion, it will be found that a siderial day consists of $23^{h}\ 56^{m}\ 4^{s}{\cdot}09$.

If the sun were exactly in the centre of the orbit of the earth, and if this orbit were an exact circle, the interval between any two successive transits of the sun would be influenced by the obliquity of the ecliptic only. But from various causes the interval between two successive transits of the sun is not uniform; therefore we take a mean of the intervals and obtain what we call 'a mean solar day.'

Of the two measurements of time, viz. the siderial and solar, the former is by far the most uniform, for the distance of the star is so great that the annual movement of the earth from one end to the other of its orbit does not appear to cause any displacement in the position of the star;* so that the conditions are nearly the same as though the earth were at rest as regards space, and rotated 366¼ times during one year.

An examination of diagram 1 will show that whilst the earth rotates on its axis, as shown by the small arrows, and moves round its orbit, as shown by the large arrow, there will be one less transit of a body within the circle than of a body outside this circle. If, however, the earth rotated in the same manner, but moved round the sun in a contrary direction to that in which it now moves, there would be one more transit of the sun than of a star; that is, one more transit of a body within the circle than there would be of a body without that circle.

As far as our investigations at present carry us, it would appear that, when we wish to ascertain what is a uniform standard of time, we have merely to refer to the successive transits of any star, and to note whether these successive transits occur at equal intervals. Taking long periods, we might inquire whether say 10,000 transits of a star now occur in exactly the same time that 10,000 transits of the

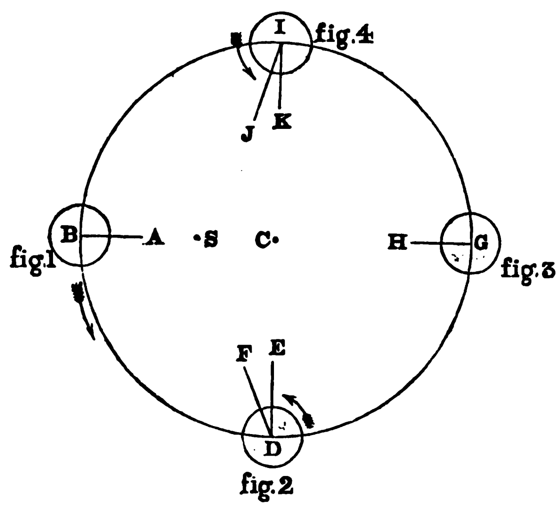
* We here omit notice of aberration, and treat only, as before remarked, of general effects.

same star occurred say five centuries ago. But to what are we to refer? We have taken the rotation of the earth as our standard, and it is only by comparisons that we can ascertain the permanence or variation of any standard. If we find a variation, it becomes a question whether it is our standard that has varied or the second item with which we have compared it. And unless we have some third object for comparison, we must still remain without any proof that we possess a uniform standard measure.

We have already referred to the earth's rotation on its axis and revolution round the sun, and have shown that the earth's revolution round the sun causes a star to pass a meridian once oftener than does the sun during an entire revolution. If the equator of the earth coincided with the ecliptic, and if the sun were exactly in the centre of the earth's orbit, then the successive transits of the sun would give us a uniform standard of time, and each solar day would be equal to any other solar day; also thirty solar days in March would be equal to thirty solar days in June. If, however, these conditions do not occur, we obtain an irregularity in the solar days, which prevents a solar day being a uniform measure of time.

We will give a demonstration of one of the effects which produces an irregularity in the solar day; and as this one is very similar to another which we shall treat of at a future page, we call particular attention to this problem.

Fig. 1 we suppose the earth at a given position in its orbit. S we will suppose the sun, which is



not the centre of the earth's orbit, this centre being at C. Fig. 1 we take as in the prolongation of the line joining C and S. When the earth is at fig. 1, both S and C would be on a meridian simultaneously. Upon the earth reaching the position fig. 2, a meridian D E would reach C, and would then rotate on to the position D F before S, the sun, came on to the meridian.

Thus, during the whole time the earth was moving from 1 to 2, the intervals of time between the successive transits of C would not be equal to the intervals of time between the successive transits of S. If we indicate the number of transits of C by n, then this interval of time would be equal to n transits of S,

minus the angle of rotation E D F, or in other words, a solar day would be a longer period of time than the interval between two successive transits of C.

Upon reaching fig. 3, in the prolongation of S and C, the meridian G H would pass C and S at the same instant, and thus during the passage of the earth from 2 to 3, S, the sun, has caught up as it were C, and now transits with it.

When, at fig. 4, the meridian I J reaches S, the sun, before it reaches C, the meridian I J would have to rotate to the position I K before the centre C came upon it. Thus S has now advanced before C, and has done so gradually during the time that the earth has moved from 2 to 3.

On reaching fig. 1 again the two would transit simultaneously.

Thus, during the passage of the earth from 1 to 3, C is always in advance of S. During the passage of the earth from 3 to 1 C is always behind S.

If, then, we take the mean of all the transits of S during an entire revolution of the earth in its orbit, this mean would be the same as the interval between two successive transits of C.

This difference between the position of C and S causes a difference between a solar day and a mean solar day. The value of the angle J I K at its greatest is about 1° 55′ 33″, and this is *one* of the causes which prevents a solar day from being an exactly uniform measure of time.

The two points, as we may term them, S and C,

form an angle when viewed from the earth, except when the earth is at fig. 1 or fig. 3, and this angle at its maximum is $1° 55' 33''$. The maximum would occur when the earth was equidistant from S and C; that is, near the positions shown by figs. 2 and 4.

If the earth travels over equal arcs in its orbit, the angle formed by S and C would not change uniformly. The rate at which this angle will vary is nearly as the sine of the angle formed by two lines, one joining fig. 1 and C, the other joining C and the earth's position.

From these complications (as we may term them), the sun is not a convenient body to employ as a means of obtaining a standard of time, and we now turn our attention to the stars in order to select a uniform standard.

In consequence of the immense distance of the stars from the earth, we may, in dealing with them as time-measures, reason in the same way as if the earth merely rotated on its axis, and did not move from one end to the other of its orbit.

If the rotation of the earth took place without any other movement occurring, the interval of time between two successive transits of any star would be equal to the time occupied by the earth in its rotation. Also the intervals of time between the successive transits of every star would be exactly equal to each other. Under such conditions, it would be of no consequence what star we selected as a datum point from which to count time, because as all stars

appeared to move round us exactly uniformly, one star would be as efficient as another.

In addition to its rotation on its axis, the earth has a third movement, which changes the direction of a meridian, and consequently alters the intervals of time at which a star transits each meridian. If the conical motion of the earth's axis be combined with that movement which we have described as similar to a second rotation, it follows that there will be one point in the heavens, and only one point, whose successive transits will give us uniform intervals of time; and this point is the centre of the circle traced by the earth's axis during one revolution of the equinoxes. If the pole of the ecliptic be the centre of the circle thus traced, then the interval between the successive transits of the pole of the ecliptic will give us a uniform measure of time equal exactly to the time occupied by the earth in rotating on its axis. If, however, there be a decrease in the obliquity of the ecliptic, it follows that the pole of the ecliptic is not the centre of the circle traced by the earth's axis; and consequently the intervals between the successive transits of this pole do not give us a uniform standard or measure of time, nor is the interval between any two successive transits of the pole of the ecliptic equal to a rotation of the earth. From these facts, it follows that the only true standard measure of time is to be obtained by the intervals between the successive transits of that point in the heavens which is the centre of polar motion,

P

provided that the conical movement of the earth's axis is combined with and produced by the slow rotation of the earth from east to west during one complete revolution of the equinoxes.

If it be supposed that the pole of the ecliptic shifts its position in the heavens, but always remains the centre of the *curve* (for the curve cannot then be part of a circle) traced by the pole of the heavens, it still follows that the successive transits of the pole of the ecliptic will not give us uniform intervals of time. Startling, then, as the announcement may be to those theorists who imagine that every item in astronomy has long since been solved, and may be disposed of, yet it is a fact, from which there is no escape, that when we come to accurately investigate this problem of time, it can be demonstrated that up to the present date no astronomer knows in what a rotation of the earth really consists, and by what phenomenon it is to be measured. Nor can the value of a rotation be known or measured until the true course traced by the earth's axis is ascertained, and the real movement is discovered which the earth makes in combination with the conical movement of the axis. It is, of course, easy to approximate, for ten or twenty years in advance, to results which can be then predicted, although the actual value of a rotation of the earth is not known, especially when it is possible to add a few minutes and seconds here and there to make theories fit into and with actual facts. Between the years 1833 and 1834, for example, 3^m 4^s of time was

fudged to make matters appear satisfactory.* Such proceedings, however, do not belong either to geometry or astronomy; and those sanguine persons who overlook these matters, and who claim that the minutest item in astronomy has long been correctly known to the hundredth of a second, are not likely to mislead any but the ignorant, or those whose faith is greater than their knowledge.

* See note at end of *Outlines of Astronomy*.

CHAPTER XV.

CHANGES IN A TERRESTRIAL MERIDIAN.

The astronomers of the past seem to have assumed that the movement of the earth that accompanied the conical movement of the axis was that which we have described in Problem 3, viz. a conical movement of the earth's axis, accompanied by a rotation of the earth from left to right. The differences between a conical movement of the axis and a conical movement of the semi-axis, it will be evident, are very great and most important.

In order to understand one of the principal effects of this supposed movement of the earth, we may describe it in the following manner:

Suppose the earth had no rotation on its axis and no *annual* revolution round the sun, and its only movement consisted in a slow revolution round the sun in the same direction and during the same time that a revolution of the equinoxes occurred. There would, owing to this movement of the earth and its axis, be one solar day, but no siderial day, during this long period, as may be shown by the following diagram.

Fig. 1 represents the position of the earth at a given date, the pole being at P, and M P R the meri-

fig. 2.

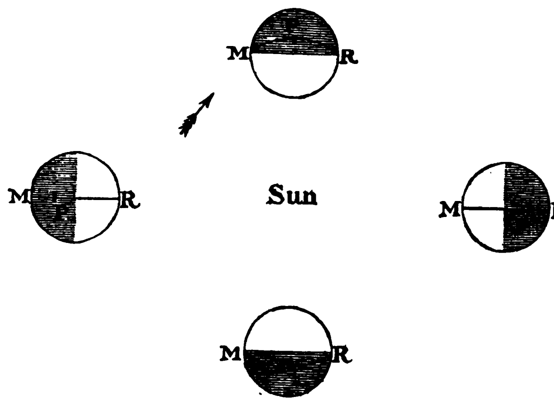


fig. 1. fig. 3.

fig. 4.

dian. The sun now shines on that part of the earth on which the meridian P R is located. In about 8000 years from the date at which the winter solstice occurred at fig. 1, it will occur at fig. 2, in consequence of the conical movement of the axis. On the assumption that the earth moves in connection with the axis in the manner described in Problem 1, the meridian M P R, fig. 2, would still be directed almost in the same manner as it was at fig. 1, and the same side of the earth that was directed towards a distant star when the earth was at fig. 1 would also be directed towards the same distant star when the earth was at fig. 2.

But the same side of the earth would not be turned towards the sun at fig. 2 as was turned towards it at fig. 1.

Carrying on the same reasoning, it will be seen that all parts of the earth would in turn be illuminated by the sun during this revolution of the equinoxes, and the effects would be that there would be one solar day during this complete movement, but no siderial day.

If, however, the rotation of the earth during the long period of the revolution of the equinoxes occur with the conical movement of the axis, and this occurred alone, then the same side of the earth would always be turned towards the sun, just as the same side of the moon is always turned towards the earth; consequently there would be no solar day and one siderial day during this long period.

Whilst, however, by the first-named problem, a solar day would occur, and the movement of the sun round the earth would be in the same direction as that which the sun appears to follow, in consequence of the diurnal rotation, viz. from east to west, by the second movement a star would appear to move round the earth from west to east, or contrary to that which it would appear to follow if the earth's rotation on its axis in 24 hours were considered.

All these problems are of grave importance as affecting the changes in stars' right ascensions, and as regards the true measure of time; for unless every detail connected with this conical movement of the axis and the resulting change in the earth itself is known, the changes depending thereon cannot be known.

It is difficult to discover on what evidence the olden astronomers believed that there was a conical movement of the whole axis, and not of the semi-axis; also why they believed that *the* movement was that of the axis only, without any rotation of the earth. That the suppositions of the olden astronomers have been taken for granted is, however, a fact; and the statements put forward more than two centuries ago have been copied in modern times; even that impossible one relative to the pole of the ecliptic being the centre of polar motion.

This problem is one requiring very careful reëxamination, because it resolves itself into separating the influence of the diurnal rotation of the earth from that of the conical movement of the axis; for a given meridian would attain, after a certain interval of time, almost exactly the same position by means of the diurnal rotation that it would attain by another movement, due only to the resulting motion connected with the conical movement of the axis.

If, then, a meridian be carried into a certain position, the question arises whether it has attained this position in consequence of the diurnal rotation of the earth on its axis, because of the conical movement of the axis, or on account of the combined influence of the two movements. In fact, until we know whether it is the earth's axis or semi-axis that makes a conical movement, whether the earth has a slow movement of rotation combining with the conical motion of the axis, or performs the conical

motion without the slow rotation, and also unless we know where the true centre of polar motion is located, we cannot state in what a rotation of the earth really consists.

Every star gives for the earth's rotation, by its successive transits, a different value for the rotation when long periods of time are taken into account; therefore, although for brief periods we may select any star and measure a siderial day by the intervals of time between the successive transits of such star, yet for long periods we cannot accept so rough and imperfect a method.

Sir John Herschel, who was perhaps better acquainted with the general principles of the measurement of time than any astronomer of his day, states in Article 909, *Outlines of Astronomy*, the following:

'Now a conical movement impressed on the axis of rotation of a globe already rotating is equivalent to a rotation impressed on the whole globe round the axis of the cone, in addition to that which the globe has and retains round its own independent axis of revolution.'

The same writer then proceeds to point out that the position taken up by a given meridian after a certain amount of polar motion is that which it would attain by our movement shown in Problem 3, viz. a conical movement of the earth's axis resulting from a slow rotation round the axis of the cone, and in a direction in opposition to that of the diurnal rotation.

The reader who has followed us in our demonstrations, and has studied the four problems on pages 180 to 186, will perceive that the statement quoted above, viz. that 'a conical movement impressed on the axis of rotation of a globe already rotating is equivalent to a rotation impressed on the whole globe round the axis of the cone,' is not necessarily correct. The effect *may* be the same as a rotation impressed on the whole globe, but it does not follow that it *is* the same. This practice of hastily assuming only one possible case as that which really occurs, when there are four movements which *may* take place, was too much the habit in former times, and it has led to much of that confusion which has now culminated in the endless tables of stars' supposed proper motions, and in the changes in measurements of time.

If the conical movement occur with the rotation, it follows that during one conical movement of the axis there will be one rotation to be accounted for in a direction opposite to that of the diurnal rotation; whereas if it does not occur the diurnal rotations will not be thus diminished by one rotation.

The effect produced upon a given meridian by the two movements of the pole will be understood by reference to diagram, p. 219, where P represents the position of the pole at any date, and R E P S a meridian.

Upon the pole P being carried to P′ by the conical movement, the meridian R E P S would assume the

position R P′ S if the conical movement were unaccompanied by the rotation of the globe from east to west; but it would assume the position M E P′ L if it were accompanied by this rotation.

The diurnal rotation is in the direction from M to R; therefore it would require a portion of the diurnal rotation to make the meridian M P′ L take the position R P′ S′. Consequently we should have singular changes as regards transits, which may be understood from the following:

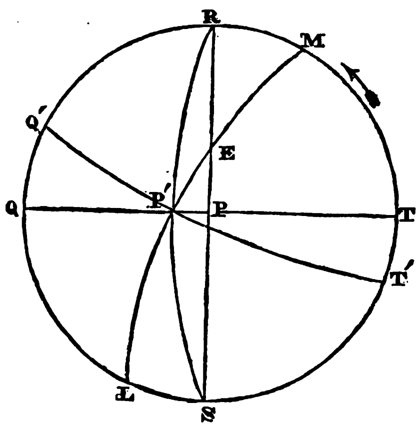
With the meridian in the position M P′ L, E would be on the meridian after the polar movement from P to P′; but with the meridian at R P′ S, E would have passed the meridian by the angle of rotation R P′ E.

Taking P P′ as 20″, we should obtain 50″ for the angle P E P′, and 50″, in terms of the diurnal rotation, is about $3^{s}\cdot 3$. If, then, we consider that 366 rotations of the earth occur whilst the pole moves from P to P′, we should have, after these rotations, the meridian in the position L P′ E M if the movement occurred according to Problem 3, and in $3^{s}\cdot 3$ afterwards the meridian would occupy the position R P′ S. If, however, the movement be that called No. 4, the meridian, after 366 rotations of the earth, would occupy the position R P′ S. Hence 366 rotations under one condition would equal 366 rotations $+ 3^{s}\cdot 3$ under the other condition.

The meridian with which we have just dealt is one which passes through the pole of the ecliptic at the start, and is then transposed either to the position

R P′ S or M E P′ L by the conical movement of the earth's axis. We will now examine a meridian that passes through the equinoctial point, and is changed in some way by the conical movement of the axis, which carries the pole over a given arc.

Let Q P T represent a meridian of 0 and 12 hours' right ascension, the pole being at P, and E the pole of the ecliptic.



Let P′ be the position of the pole after a given period, this position being attained by movement No. 4. The meridian Q P T would be transferred to the position Q P′ T by this movement, this new position being but slightly different from that formerly occupied by this meridian.

If, however, the conical movement of the axis occurred as described in No. 3, the meridian Q P T would, by this movement alone, be carried into the position Q′ P′ T′; and to occupy the position Q P′ T, this meridian would have to be carried by the diurnal

rotation over the angle Q P′ Q′. Let us take this angle at 30°. We should, by one movement, have the meridian Q P T two hours in advance of what it would be by the other movement; an item very large when minute accuracy is required.

Thus, whilst the centre of the earth would occupy exactly the same position in both cases, as regards the sun and a star, the sun and the moon, or a point on the ecliptic, and the pole and axis would also occupy exactly the same position in both cases, yet by one case a meridian would be two hours more forward than by the other; or in other words, a portion of a diurnal rotation would have been completed in one case and not in the other.

It will be evident, from even a brief examination of this problem, that it is one of great importance, not only as a means by which to obtain the exact measure of the earth's rate of rotation, but also on account of certain phenomena by which endeavours have been made to test or ascertain the truth of historical events. In consequence of the *centre* of the earth being in exactly the same position in its orbit, no matter which movement of the earth accompanies the conical movement of the axis, it follows that any event which occurred say about 400 B.C., and due to the earth's *centre* being in a certain part of its orbit, would act differently on a certain locality on the earth's *surface*, according as the one or the other of these movements occurred.

For example, supposing a total eclipse occurred

at say 400 B.C., and was total at Q on the sphere Q′ Q S R, diagram, p. 219, the question would naturally arise, What meridian was then in the position P′ Q? Was it the meridian that would be there by the movement called No. 3 or that called No. 4? If we assumed that the movement was that called No. 3, then the meridian on which this eclipse was total would be 30° in longitude to the east of the meridian P′ Q′; whereas if the movement were that called No. 4, the eclipse would be total on the meridian P′ Q′, which would then be in the position occupied by P′ Q.

Thus we cannot state where an eclipse of the sun was visible in very ancient times, unless we know which of these two movements of the sphere accompany the conical motion of the earth's axis; and we must therefore first endeavour to decide which is that most probable or most certain.

In order to investigate this problem, we will consider the effects which must occur in consequence of either of the two movements during long intervals of time, and endeavour to keep this movement independent of all others that affect the earth.

Referring to diagram, p. 219, it will be seen that if the meridian S P E R be transferred by the conical movement of the axis to the position L P′ E M, the point E would always remain on this meridian, the displacement even during an entire revolution of P round E never taking E from this one meridian.

If the meridian S P E R be displaced by the conical

movement to the position S P′ R, the point E is immediately displaced from this meridian, and in order to bring E on a meridian, an observer must travel, or be carried by a diurnal rotation, to the meridian P′ L.

Here, then, is a most marked difference, which may enable us to decide which of these two movements occurs.

We will now bring to our aid a comparison standard, and endeavour to test by this standard something in connection with the rotation of the earth. We will take as our standard a clock, and at the instant when our meridian is in the position S P E R, diagram, p. 219, we will start this clock to indicate 18 hours.

First, let us suppose that no movement whatever except that produced by the diurnal rotations occurs in the pole P or the meridian S P E R, and that the earth rotates uniformly; also that every time the clock indicates 18 hours the point E is on the meridian. We should have a standard of time by this, and could note what effects are produced by the conical movement, provided we can depend on our standard clock.

Next, let us suppose that the meridian R P S is, by the action of the conical movement, carried to M E P′L. During and due to this movement, E is never removed from this meridian, and therefore will always be on it at the instant that the clock shows 18 hours, just as would be the case if neither the clock nor the meridian moved.

If, however, the meridian be transferred to the position R P′ S′, E is removed from the meridian, and would have been on it before 18 hours was indicated by the clock if a rotation also occurred, the interval before being measured by the angle of rotation S P′ L.

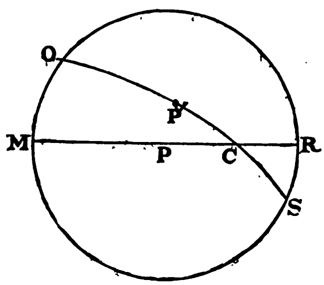
It will be evident that, by the first-named movement of the meridian, we should obtain a uniform standard or measure of time by the successive transits of E, this standard being the same in length as the time occupied by the earth in rotating; whereas by the second movement the successive transits of E would not be uniform with the rotation of the earth, though the interval between any two successive transits would be of almost a constant value. But the only conditions under which the successive transits of E would be uniform are that E is the pole of the circle described by the earth's axis in the heavens.

Having examined the various problems in the preceding chapter, we are in a position to speak of a subject of the utmost importance to astronomy, which is one that has been long ago supposed to have been solved. This supposition, however, has been arrived at merely because the olden astronomers were not acquainted with the geometrical laws connected with the conical movement of the earth's axis. The problem which we now purpose examining is, What is a rotation of the earth, and by what is it measured?

It will be evident to the reader that, before we can decide on what is a rotation of the earth, we must

know which is *the* conical motion of the axis of the earth that really occurs. In the following diagrams we demonstrate the varied results on a given meridian, according as one or other movement takes place, and show what variations occur in connection with certain other matters.

In annexed fig. we take a meridian, M P C R, as a given meridian, and P the pole. First, we will suppose that no movement occurs in the earth's axis; then, after say 366 rotations of the earth have been completed, the meridian would again occupy the position M P C R.



Next let us suppose that the pole P has been carried to P′ during 366 rotations of the earth on its axis, and that, combined with this movement of the pole, the earth has slowly rotated round a second axis in the manner shown by Problem 3. The meridian, after 366 diurnal rotations of the earth, would then be in the position O P′ C S, the angle P′ C P being dependent on the value of the arc P P′ and the arcs P′ C and P C. The actual precession of the equinoctial point will be due to the length of the arc P P′ and to the value of the obliquity at the time; *therefore the actual precession of the equinoctial point would be the same in both the cases referred to as Problem* 3 *and Problem* 4, no matter whether the slow rotation of the

earth was or was not combined with the conical motion of the axis.

The radius of polar motion being $29° 25' 47''$, and the annual arc moved over by the pole being $20''\cdot158$, it follows that, according to movement 3, the earth during each year would perform part of a rotation from east to west—that is, in opposition to the diurnal rotation—and round an axis the pole of which was at C, fig. 1, page 224; the amount of which rotation can be found from the equation

$$\frac{20''\cdot158}{\sin. 29° 25' 47''}$$

which gives for the annual arc of rotation $41''\cdot03$.

Taking the obliquity of the ecliptic as $23° 27'$, we should have for the annual precession

$$\frac{20''\cdot158}{\sin. 23° 27'}$$

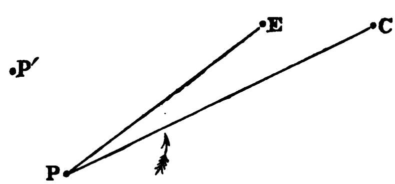
which gives about $50''\cdot3$ for this rate.

We have, then, for the actual arc of the ecliptic intercepted between two successive equinoxes $50''\cdot3$, due to the change in the position of the pole, and to the obliquity of the ecliptic, but entirely independent of the position of the centre of polar motion, and of the movement of the earth that accompanies the conical motion of the axis. Also we have an apparent rotation of the earth from west to east, at the rate of $41''\cdot3$ per annum, round an axis directed to the centre of polar motion.

It is evident that a point C on the sphere will be reached by the meridian after exactly one diurnal

rotation of the earth has been completed, no matter whether or not a movement of the pole occurred; therefore we should have an exact measure of the rate of the earth's rotation by the interval of time which elapsed between two successive transits of the centre of polar motion. So also we should have during an entire revolution of the equinoxes just as many transits of the centre of polar motion as there were rotations of the earth on its axis; and the intervals of time between these transits would give us a standard or uniform measure of time, which would be unchangeable as long as the rotation of the earth was uniform, and the centre of polar motion fixed on the sphere of the heavens.

It would follow from the above that the successive transits of any point in the heavens within the circle actually described by the pole, and nearer the pole than is the centre of polar motion, would give a measure of time *less* than the time occupied by the earth in rotating on its axis. The pole of the ecliptic, for example, is at present nearly 6° nearer the pole of the heavens than is the centre of polar motion; the relative positions on the sphere of the heavens being much as shown in the annexed diagram, where



P is the position of the pole of the heavens, E the

position of the pole of the ecliptic, and C the centre of polar motion. A meridian such as P C would have the point C on it; then this meridian, being carried by the diurnal rotation to E, would cause E to transit after C by an interval of time measured by the hour angle E P C. When the pole of the heavens had been carried to P′ by the conical movement of the axis, E and C would transit simultaneously, because C E P′ is a meridian. Consequently E has transited the meridian a given number of times, and has also moved over an angle E P C during the same interval of time that C has transited only the same number of times. If, therefore, we measured rotations of the earth by the interval between successive transits of E, we should be estimating this rotation, not only as less than it ought to be, but the rate itself would not be uniform.

Such a movement of the earth as that just referred to would afford us a very simple uniform standard of time, when we knew where the centre of polar motion was located; but we should not be able to indicate in what a rotation of the earth actually consisted unless we knew where this centre was located.

Up to the present time the position of the centre of the circle traced by the pole has been supposed to be coincident with the pole of the ecliptic, and the successive transits of the pole of the ecliptic have been supposed to give a uniform measure of time. This supposition is incorrect, and has led to erroneous conclusions in connection with events sup-

posed to occur and recur after certain intervals of time.

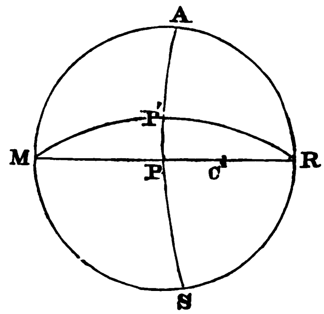
We have, then, by this movement of the earth, combined with the conical motion of the axis, a standard measure of time by means of the successive transits of the centre of polar motion, the interval between these transits being exactly the same as the time occupied by the earth in rotating on its axis. We also have an apparent rotation of the earth during a long period of time, and from east to west, round an imaginary axis directed towards a point in the heavens distant 6° from the pole of the ecliptic.

The change in direction of the earth's axis, and the course traced in the heavens by the pole of the earth's axis, may be taken as settled questions. Geometry demonstrates that the course of the pole is round the circumference of a circle the centre of which is 6° from the pole of the ecliptic, and this course gives for the climatic changes on earth exactly those which geology proves have occurred. Consequently we need not at present refer further to the actual changes occurring to the earth's axis, but we have to refer to the changes produced on various meridians and various zeniths by that same movement which causes the change in direction of the earth's axis, or more correctly speaking, of the semi-axis.

We have seen that by the last movement described the earth rotates once from east to west round the centre of polar motion during one revolution of the equinoxes. We will now describe the

movement of the earth which *may* occur in combination with the axial motion, and which is not a rotation.

In the annexed diagram P represents the pole of the heavens, C the centre of polar motion, M P C R a given meridian. After a certain interval of time, the pole P we will suppose carried to P′, and accompanied by and due alone to this movement of the axis and pole P; we will suppose the meridian M P C R transferred to M P′ R. Such a change in the meridian would be that described as resulting from the movement Problem 4. Such a movement as the above would cause C, the centre of the arc P P′, to be removed from this meridian, and to be on a meridian west of M P′ C. When, then, the diurnal rotation is considered, and this diurnal rotation accompanies the above movement, the point C, instead of transiting when each siderial rotation of the earth is completed, will transit earlier by an interval of time measured finally by the hour angle R P′ C.



Instead of the successive transits of C giving us a uniform standard of time equal exactly to the time occupied by the earth in rotating on its axis, these transits would give us an interval of time less than that of one rotation of the earth on its axis; whilst a point near R or M would give us more nearly, by their

successive transits, an interval of time equal to that occupied by the earth in rotating.

By this latter movement of the earth no point on the earth's surface would remain at rest during any part of the entire conical motion of the axis, and as affected only by this conical motion, and independent of the diurnal rotation; but by the last-named movement, viz. Problem 3, there would be a point, viz. the centre of polar motion, fixed during the conical movement, if the earth were not affected by the diurnal rotation.

In order to make the differences resulting from these two movements as intelligible as possible, we may give another illustration by supposing the movement to be transferred to the heavens themselves. Then, assuming the diurnal rotation to go on regularly, the heavens slowly rotated in 31,580 years round the centre of polar motion, and from right to left. This would give the effects referred to in Problem 3.

If, instead of the heavens rotating, the centre of polar motion described a circle round the pole of the heavens, and all other stars also described circles round a series of points in the heavens, then the movement would be similar to that described as No. 4 Problem.

When we consider that observations of fair accuracy have been carried on for at least 100 years as regards the general and annual changes in the right ascension and declination of stars, it is evident that

we have ample data on which to frame conclusions relative to which of these two movements is the more probable. As, however, there are other important variations in the two movements which have yet to be examined, we next turn our attention to one of these, viz. the changes that will be produced in a given zenith by the movement of the earth combining with the conical movement of the earth's axis.

CHAPTER XVI.

CHANGES IN A GIVEN ZENITH.

The zenith of a locality on earth 38° 32′ from the pole of the earth, therefore in latitude 51° 28′, is by the diurnal rotation carried round a circle of the sphere, which circle contains 224°·2, found from the equation $360 \times \sin. 38° 32' = \text{number of degrees}$.

The zenith of latitude 51° 28′ would, at the period of the winter solstice, at midnight, be distant from the pole of the ecliptic 38° 32′ + the value of the obliquity. Taking the obliquity at 23° 28′ (in round numbers), the zenith of latitude 51° 28′ would, at the date of the winter solstice, be 62° from the pole of the ecliptic at midnight, and $38° 32' - 23° 28' = 15° 4'$ from the pole of the ecliptic at midday.

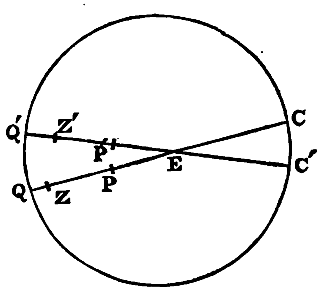
After 366 complete siderial rotations of the earth had occurred, the same zenith would occupy exactly the same positions in the heavens that it occupied at first, provided there were no conical or other movement of the axis of the earth. The question now is, What position will the same zenith occupy after 366 rotations of the earth have occurred, and taking into account that movement of the earth which really occurs in connection with the conical motion of the axis?

If, as has been erroneously supposed, the earth's axis traced a circle round the pole of the ecliptic as a centre, the zenith would, whilst it maintained a constant angular distance from the pole of the ecliptic at the termination of each rotation, be carried over an arc the value of which would be found as follows:

Taking the polar movement in arc as $20''\cdot15$ annually, and the obliquity as $23° 28'$, the apparent rotation of the sphere of the heavens would amount to

$$\frac{20''\cdot15}{\sin. 23° 28'} = 50''\cdot3$$

That is to say, the angle at the pole of the ecliptic formed by two arcs, one joining the pole of the ecliptic and the pole of the heavens at a given date, the other joining the pole of the ecliptic and the pole of the heavens when 366 rotations of the earth were completed, would be $50''\cdot3$. Now this angle of $50''\cdot3$ is the amount of the precession due to a polar movement of $20''\cdot15$, as may be shown by the following diagram, where E represents the pole of the ecliptic, P the pole of the heavens at a given date, E P therefore a portion of the solstitial colure, P′ the position of the pole after 366 rotations of the earth, E P′ the new position of a portion of the



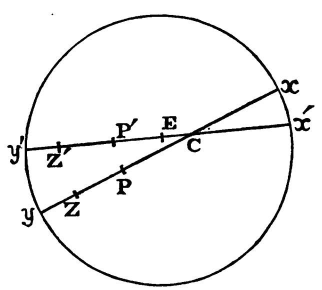
solstitial colure after 366 rotations of the earth; P′E P the angle representing the value of the precession

due to a polar movement of $20''\cdot15$, with an obliquity of $23°\ 28'$.

The zenith Z would by this movement be carried to Z′; the length of the arc Z Z′ can be found by the equation $50''\cdot3 \times \sin.$ E Z or E Z′, the values of E Z and E Z′ being equal.

$EZ = PE + PZ = 23°28' + 38°\ 32' = 62°$; consequently $Z\,Z' = 44''\cdot4$; and this is the amount of arc measured on the sphere over which the zenith of a latitude of $51°\ 28'$ would be carried during 366 rotations of the earth, and due to the pole of the ecliptic being the centre of polar motion, and an apparent rotation of the sphere occurring with the polar motion.

As we have already demonstrated, the pole of the ecliptic is not the centre of polar motion; this centre is 6° from the pole of the ecliptic, and so situated, that at the period of midnight at the winter solstice the zenith of $51°\ 28'$ would be distant from the centre of polar motion about 68°, found in the following manner:



C represents the centre of polar motion, E the pole of the ecliptic, P the pole of the heavens.

Then, as before, Z, the zenith, will be $38°\ 32'$ from P, and P is $29°\ 25'\ 47''$ from C. Therefore $Z\,C = 67°\ 57'\ 47''$.

In order to find the value of the arc passed over

by the zenith Z during 366 rotations of the earth, we must find the greatest apparent rotation of the sphere, which is to be obtained as follows:

$PP' = 20''{\cdot}158$; $CP = 29° \ 25' \ 47''$. Therefore, the great circle of the sphere 90° from C would, for a movement of 20″·158 distant 29° 25′ 47″ from C, be carried over an arc of

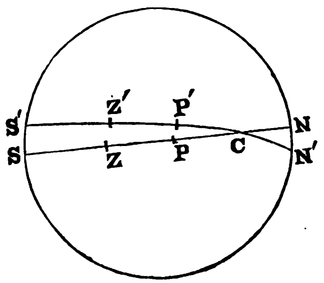
$$\frac{20''{\cdot}158}{\sin.\ 29° \ 25' \ 47''} = 41''{\cdot}03$$

The zenith Z would be carried over an arc of $41''{\cdot}03 \times \sin.\ CZ = 37''{\cdot}9$ whilst the pole P was moving 20″·158.

For exactly the same amount of movement of the pole of 20″·158 during 366 rotations of the earth, the zenith of Greenwich, which is in latitude 51° 28′, would be carried over an arc of 44″·4, if the pole of the ecliptic were the centre of polar motion, and of 37″·9 in consequence of this centre being 29° 25′ 47″ from the pole of the heavens. That is to say, the zenith of Greenwich would occupy a position in the heavens 6″·5 distant from the point it would occupy if the pole of the ecliptic were the exact centre of polar motion.

A point on the southern horizon at midnight at the winter solstice would be 90° from the zenith, consequently 157° 57′ 47″ from the centre of polar motion. The point on the southern horizon would be therefore 67° 57′ 47″ from the great circle of the sphere 90° from the centre of polar motion, and this point would be carried over an arc of $41''{\cdot}03 \times \sin.\ 22° \ 2' \ 13'' = 15''{\cdot}3$.

The point on the horizon on the northern meridian would also be 22° 2′ 13″ from the centre of polar motion, and would be carried over an arc of 15″·3 during 366 rotations of the earth, and due to the conical movement of the earth's axis round a centre 29° 25′ 47″ from the pole of the heavens. The displacement of the meridian therefore due to the polar movement, and during 366 rotations, would be, for a meridian of 6 and 18 hours' right ascension, as follows:



That part of the meridian passing through C, the centre of polar motion, would remain in the same position as though the earth rotated round a fixed axis, the direction of which never changed.

P, the pole, would be carried to P′, the arc P P′ being 20″·158.

Z, the zenith of Greenwich, in latitude 51° 28′, would be carried to Z′, the arc Z Z′ being 37″·9.

S, the southern point on the horizon, would be carried to S′, the arc S S′ being 15″·3, the same in value as N N′.

The direction of the diurnal rotation is opposite to that in which the zenith Z has been carried by the polar motion, and all parts of the earth have been carried in the opposite direction to that in which they would be carried by the diurnal rotation, except

those parts situated between C and P. It would follow, therefore, that a star situated in the heavens, and in the zenith of N, would not come on the meridian S′ Z′ P′ N′ until the diurnal rotation had carried N′ to N. Nor would a star in the zenith of S be on the meridian S′ Z′ P′ N′ until the diurnal rotation had carried the meridian P′ S′ to P′ S. At the same time, all stars in the triangle P′ C P would have passed the meridian below the pole, during the 366 rotations of the earth, 366 times + a small arc of rotation depending on the angular distance of such stars from the point C and the arc P′ C.

The point C therefore remains, as we may term it, a fixed point, uninfluenced by the polar motion. The intervals of time between the successive transits of the point C, or of a star situated in the heavens as is C, would be the same, no matter whether the pole P changed its position to P′, or remained fixed at P. The intervals between the successive transits of C would therefore give us a uniform measure of time, which would be constant for all time, provided the centre of polar motion remained fixed in the heavens, and the earth's rate of rotation were uniform.

The intervals between the successive transits of any other point in the heavens would not give us a uniform measure of time, because these points would be brought on to a meridian more rapidly or more slowly than they would be if the earth's axis did not change its direction. But the increase or decrease in the intervals at which certain points in the heavens

did transit a meridian can be calculated, but this calculation can be made only when the true position of the centre of polar motion is known. As up to the date of the publication of our last work, when the position of the centre of polar motion was demonstrated, the true centre was not known, but was supposed to be the pole of the ecliptic, it follows that up to that time the true value of a rotation of the earth was not known, and the true method of measuring what this rotation was had not been recognised.

At the instant that the sun's centre and the pole of the ecliptic transit a meridian simultaneously, at that instant the solstice occurs. If the sun's centre and the pole of the ecliptic be on the same side of the pole of the earth, then the winter solstice occurs to the northern hemisphere; whereas if the sun transit the meridian on the opposite side of the pole to that on which the pole of the ecliptic transits, then the summer solstice occurs to the northern hemisphere.

If the earth's axis always pointed in exactly the same direction, and the earth rotated at a uniform rate, the successive intervals of time indicated by successive transits of the pole of the ecliptic would be all equal to each other. Also the interval of time between the 1st and the 366th transit of any one star would be equal to the interval of time between the 1st and the 366th transit of any other star, no matter in what part of the heavens these stars were located.

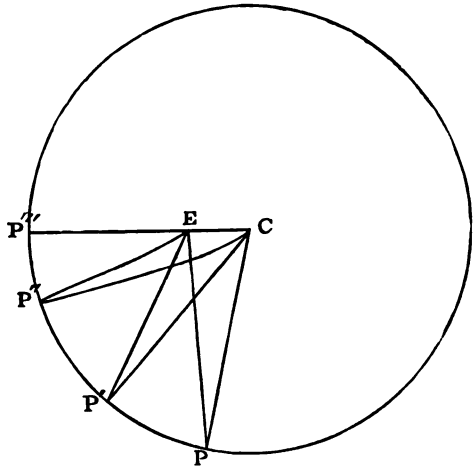
In consequence of the pole of the heavens changing its direction about $20''{\cdot}158$ annually, the inter-

vals of time between the successive transits of the pole of the ecliptic are not equal, and the interval of time between the 1st and the 366th transit of the pole of the ecliptic is not the same during the year 1873 that it was in 1872; nor is this interval the same in any one year that it was in the year preceding it, or that it will be in the year following, the only uniform measure of time being the intervals between the successive transits of that point in the heavens at which the centre of polar motion is located.

It being a fact that the interval of time between the 1st and the 366th transit of the pole of the ecliptic at any one date gives us a measure of time which is not equal to the interval between the 1st and the 366th transit of that pole at another date, leads to most important results connected with the length of the tropical year, the amount of arc intercepted between two successive solstices, and the true value of the earth's rotation on its axis.

In order to find the differences between the intervals of transit of the pole of the ecliptic at former dates and at the present time, it is necessary to refer to the relative positions of the pole of the ecliptic, the pole of the heavens, and the centre of polar motion, and to note the changes resulting from the apparent travelling of the pole of the heavens round the centre of polar motion, and to calculate the variations due to this movement. These variations may be described as follows:

C represents the position of the centre of polar motion, E the position of the pole of the ecliptic,



P''' the position of the pole of the heavens at the date 2295·5 A.D., P the position of the pole of the heavens when the arc E P is at right angles to the arc joining E and C, P P' P'' P''' the course traced by the pole of the heavens round C as a centre.

From data already given, we know that E C = 6°, P C = 29° 25′ 47″; and as the triangle E C P may be taken as a right-angled spherical triangle, we have the side E C = 6°, the hypothenuse P C = 29° 25′ 47″, to find the angle C P E.

The angle C P E (omitting seconds) would be 12° 17′. In order to find the value of the angles C P′ E and C P″ E, we may *approximate* to these angles by the simple formula 12° 17′ × sin. P‴ E P″ = E P″ C; also 12° 17′ × sin. P‴ E P′ = E P′ C; and so on.

Let us take P‴ E P′ as 60°, then E P′ C = 10° 38′, and from this data we can demonstrate the decrease which has occurred in the interval between the transit of C and of E, the pole of the ecliptic, during the movement of the pole of the heavens from P to P′.

When the pole was at P and a given meridian at P C, the point C would be on the meridian. The rotation of the earth being from right to left, the meridian P C would, by the diurnal rotation, be carried to P E. The angle at P has been shown to be 12° 17′; therefore, converting this angle into time, we obtain $49^m\ 8^s$ for the interval of time between the transit of C and the transit of E, the pole of the ecliptic.

When the pole of the heavens is located at P′, the meridian P′ C will, by the diurnal rotation, be carried to P′ E. The angle E P′ C is, as has been shown, 10° 38′, which angle, converted into time, is $42^m\ 32^s$. The point E, therefore, will transit after C, when the pole is at P′, by an interval of time of $42^m\ 32^s$; whereas when the pole was at P, the interval between the transit of E and C was $49^m\ 8^s$. The interval therefore has decreased $6^m\ 36^s$.

The point C, by its successive transits, gives us a measure of the earth's rotation on its axis; this measure being uniform, the successive transits of the point E do not therefore give us a uniform measure of time, for during the movement of the pole from P to P′ the point E has closed on to C by $6^m\ 36^s$; so that taking N as the number of transits of C during the movement of the pole from P to P′, N′ as the

number of transits of E, then $N = N' + 6^m\ 36^s$. That is to say, N′ transits of E, the pole of the ecliptic, are less than N′ rotations of the earth by $6^m\ 36^s$.

If we take P″ as the position of the pole of the heavens at the date 1873, P‴ as the position the pole will occupy at the date 2295·5 A.D., then, referring the polar movement to a great circle, the angle P‴ E P″ would represent the precession during the interval between 1873 and 2295·5 A.D., that is, during 422·5 years. Taking the rate of the precession as 1° in 72 years (in round numbers), we obtain for the angle P‴ E P″ 5° 54′.

Taking the approximate method already mentioned, we obtain for the angle E P″ C 1° 15′, found from the equation 12° 17′ + sin. P‴ E P″. That is to say, in 1873 the pole of the ecliptic transits after the centre of polar motion about 5^m in time.

If we refer to the date 1800, we find that by the above formula we obtain for the angle E P″ C, taking P″ as the position of the pole at 1800, 1° 28′; consequently at the date 1800 the pole of the ecliptic transited after the centre of polar motion about $5^m\ 52^s$ in time. Hence during the interval between 1800 and 1873 A.D., the number of transits of E, the pole of the ecliptic, give a less interval of time than the same number of rotations of the earth on its axis by 52 seconds. It follows from the preceding that in 72 years there is a loss of time of 52^s, and equal to $0^s{\cdot}72$ per year. That is to say, 366 transits of the pole of the ecliptic give an interval of time less

by $0^s{\cdot}72$ than the interval indicated by 366 rotations of the earth on its axis. This difference will be shown *about* the mean date between 1800 and 1873; but it will not be a uniform rate, there being a second difference to be accounted for, as is the case in connection with other similar problems.

At the period of the winter solstice, when the sun's centre and the pole of the ecliptic transit simultaneously, we will suppose a clock to be started and to indicate 24 hours during an exact rotation of the earth on its axis. On the pole of the ecliptic coming to the meridian the 366th time, the clock would indicate $0^s{\cdot}72$ short of 24 hours, and would thus appear to have lost time when compared with the transits of the ecliptic pole; and if the true position occupied by the ecliptic pole relative to the centre of polar motion were not known, the clock would be supposed to have a losing rate.

We have shown that between 1800 and 1873 the angle subtended at the pole between E and C decreased 13′. We will now examine the decrease between the years 1000 and 1073 A.D. of the angle subtended at the pole between these two points, and it will be found that at A.D. 1000 this angle was 3° 47′, and at 1073 about 3° 35′; and if these calculations be worked out in all detail, it will be found that the rate of decrease is not uniform, as will be evident to any geometrician, the rate at which the point E gains on C being greatest the nearer the pole P comes to E, and the rate being least when P is farthest re-

moved from E. Consequently, as long as there is a decrease in the obliquity, the rate increases, and when an increase occurs the rate decreases.

If we had any celestial body, such, for example, as the moon, which during its revolution round the earth came into conjunction with E, or with a point within the earth's orbit, represented by E, projected on to the ecliptic, it would follow that the moon would come more and more rapidly into conjunction with E, just as the meridian comes more and more rapidly into conjunction with this point; and if we judged of the moon's rate by the recurrence of its conjunctions with this point, we should conclude that there was an acceleration of the moon's motion, whereas such would not be a fact. We now come to a problem in practical astronomy which is of the gravest importance, because hitherto the details of this problem have not been thoroughly thought out.

The tropical year is defined as the interval of time between the passage of the sun's centre from south to north of the equator, and his next passage from south to north of the equator. From observations made at remote dates being compared with modern observations, it was concluded by Delambre that the length of a mean tropical year was

$$=365{\cdot}25 \text{ days} \frac{360^\circ}{100 \times 360^\circ + 45' \, 45''} = 365 \text{ days } 5 \text{ hours } 48\text{m. } 51{\cdot}6\text{s.}$$

If we omit notice of those irregularities in the apparent motion of the sun and of the equinoctial point which are not concerned with or affected by the

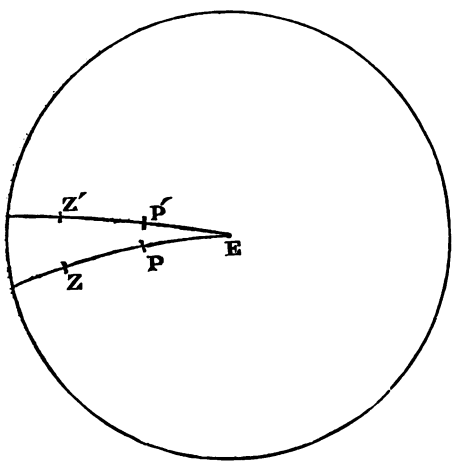
conical motion of the earth's axis, we may, as before, define the tropical year as the interval of time between two successive passages of the sun's centre from south to north of the equator. There is, however, another method by which the length of a tropical year might be ascertained, which may be defined as follows:

When the pole of the ecliptic and the sun's centre transit a meridian simultaneously, at that instant the winter solstice occurs, and this event happens about the 21st December each year. If, then, we take the interval of time between *two successive simultaneous transits* of the pole of the ecliptic and of the sun's centre, we obtain the interval of time between two successive winter solstices.

In order to prevent any mistake as regards this problem, we will deal with it as though the sun were exactly in the centre of the earth's orbit, and as if the earth's orbit were an exact circle; we can then demonstrate what results must follow and be dependent entirely on the change in direction of the earth's axis, and the corresponding change in various meridians, but independent of the eccentricity of the earth's orbit.

We must first call attention to the fact that the precession of the equinoctial point is due solely and entirely to the change in direction of the earth's axis, that the annual amount of the precession is dependent solely on the value of the obliquity of the ecliptic and on the amount and direction of the polar move-

ment, and we can then demonstrate how very important an item has hitherto been entirely overlooked by astronomers; and we would earnestly call the attention of all geometricians to this problem, for on it depend some other items of immense importance to astronomy. In the annexed diagram let E represent the pole of the ecliptic, P the pole of the heavens at a given date; the arc P E consequently represents the value of the obliquity of the ecliptic, as it is the

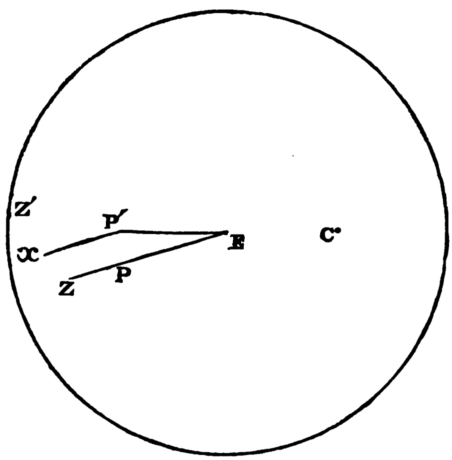


angular distance between the poles of the equinoctial and ecliptic. Let us assume for the present that the pole P moves to P′ round E as a centre, P′ E will then equal P E, and P′ will be the new position of the pole of the heavens, and the angle P E P′ will represent the exact value of the precession of the equinoctial point produced by the movement of P to P′.

Let Z be the zenith of a locality on earth, P Z

therefore will represent the colatitude of the locality of which Z is the zenith, and P Z will be a part of the meridian of Z E. On P being carried to P′ round E as a centre, Z will be carried to Z′ also round E as a centre, and the angle Z′ E Z is the same as P′ E P. The angle Z′ E Z represents the precession, as we may term it, of the solstitial colure; whilst P′ E P represents the precession of the equinoctial point, and P E P′ = Z E Z′.

In the following diagram let E be the pole of the



ecliptic, P the pole of the heavens at a given date, P E the obliquity, and let the pole P move to P′, the arc P P′ being exactly the same in value as in the last-named case, and the angle P′ E P exactly the same in value as in the last case. We have, then, P E P′ representing the precession of the equinoctial point due to the movement of P to P′, and equal exactly to what it was in the former case.

Instead of the pole P moving round E as a centre, let it move round C as a centre. There would now be only one indication that E was not the centre of the arc P P′, viz. that P E and P′E were not equal. The actual angle P E P′ would be the same, no matter whether E or C were the centre of the arc P P′; therefore the precession of the equinoctial point would be the same, no matter whether E or C were this centre, because the angle P′E P represents the precession due to the movement of the pole from P to P′. The zenith Z would not, however, by this movement of the pole be carried to the same position as in the last case, when E was the centre. The zenith Z would now be carried only to X, and would not be found on the meridian or great circle of which P′ E was a part. Z would be carried only to X, and would not be moved to Z′ on the great circle E P′ Z′. The angle X E Z is not, therefore, equal to P′E P; or, in other words, the precession of the equinoctial point is not equal to the precession of the solstitial colure, except under one solitary condition, viz. that the pole of the ecliptic is the centre of the circle described by the pole of the heavens; but as this condition does not exist, it follows that there is a difference to be accounted for.

The movement of the pole of the heavens being from left to right, and of the zenith from left to right, it follows that the point X (last diagram) is in advance of Z′ when referred to a diurnal rotation. Consequently the meridian occupied the position

Z′ P′ before it occupied the position X P′, and as when E was taken as the centre of polar motion, the meridian occupied the position P′ Z′ after a certain movement of the pole from P to P′, whereas this same position is now attained *before* the pole has moved the distance P P′, the interval before being measured by the angle Z′ P′ X of diurnal rotation.

It follows from the above demonstrations that the interval of time between two successive passages of the sun's centre from south to north of the equator is not the same in value as the interval between two successive simultaneous transits of the sun and the pole of the ecliptic. In other words, it follows that the fact of the pole of the ecliptic not being the centre of polar motion causes the interval of time between two successive equinoxes not to be the same as the interval between two successive winter solstices; and this difference in the intervals is *not due either to an eccentricity of the earth's orbit or to any change in the position of the apogee, but is produced entirely by the fact that the pole of the ecliptic is not the centre of the circle traced by the pole of the heavens.*

The problems resulting from this fact are numerous and important, and require considerable care in working out. When, however, we know the position of the centre of polar motion, they can be readily solved, and the variable results calculated. So important are the questions depending on this problem, that we will give another illustration of it, in order that there should be no mistake about the principles

involved in it. At the present time it is known that the pole of the heavens changes its position in the heavens about 20″·1 annually. It is also known that this arc of 20″·1 is traced somewhere in the direction of the first point of Aries, and therefore laterally as regards the pole of the ecliptic. It has hitherto been supposed to trace a circle round the pole of the ecliptic as a centre, but there is direct evidence that it does not trace such a circle round such a point as a centre.

Let us now assume a case, and endeavour to find by what means we could discover that such a case was really that which existed.

Let E, diagram, p. 247, represent the pole of the ecliptic, P the pole of the heavens, P E therefore the obliquity. Let the pole P move 20″·1 to P′, and let the arc P P′ be traced round C as a centre, and let the radius P C = 90°. The arc P E would now be slightly greater, slightly less, or equal to P′ E, according to the position of C, and the angle P′E P would represent the precession due to the polar movement from P to P′.

Let Z be the zenith of a locality in latitude 51° 28′. Therefore P Z = 38° 32′. Let Z P E be part of a great circle. On the pole P being carried to P′ round C as a centre, the zenith Z, influenced by the same movement, would be carried over an arc Z X round C as a centre; the value of the arc Z X would be 20″·1 × cos. 38° 32′ = 15″·6, and this would be the total displacement of the zenith of 51° 28′ due to the

movement of the pole over an arc of 20″·1 round C as a centre. Now the movement of the pole from P to P′ over an arc of 20″·1 round C as a centre would produce exactly the same amount of precession of the equinoctial point as would occur by the movement of the pole from P to P′ round E as a centre, but the results on the zenith Z would be very different in the two cases. What these differences are, and what details have to be altered, are subjects easily demonstrated. To treat of all these minute, though important, questions now, when the present astronomical authorities are under the impression that the pole of the ecliptic is, and always has been, the centre of polar motion, and that everything is known on this subject, would be as premature as to have shown the astronomers of 2000 years ago how to calculate an eclipse, when they believed the earth was a flat surface, and neither rotated on its axis nor revolved round the sun. We may, however, point out that there must necessarily be discordances somewhere between such a movement as that just described, *or any modification of it*, and a movement of the pole P and the zenith Z round E as a centre. The discordance would not be found by any marked changes in the precession, for this quantity would be the same in each case. As, however, the zenith and the meridian of each locality would be annually displaced in a very different manner, according as the pole of the ecliptic or another point was the centre of the arc traced by the earth's axis, it is evident

that discordances will occur in the meridian transits of celestial objects, in the zenith distances of various objects, and in the measurements of time. It is therefore with regard to these subjects that discordances will be found when theorists suppose that the pole of the ecliptic is the centre of polar motion, and that the entire revolution of the equinoxes occur in 25,868 years, when observation and recorded facts, as well as calculations, prove that such an assumption is not true, and the movement does not really occur.

As a problem, it may be interesting to inquire by what observations and by what facts we could discover that the earth's axis traced a circle round some other point as a centre than the pole of the ecliptic. We should not discover that such a movement occurred by any changes in the annual value of the precession, because that would be the same in amount, no matter whether the polar displacement was due to the centre being coincident with or at some distance from the pole of the ecliptic. We should or ought to know that the arc traced by the pole of the heavens was not round the pole of the ecliptic as a centre immediately we found that the obliquity of the ecliptic or angular distance between these poles was not a constant quantity; and we might suspect the pole of the ecliptic was not the centre of the arc when discordances of an unaccountable nature occurred in the right ascension and declination of celestial bodies.

Instead of this problem relative to the polar movement being one about which every item has long since been known—a belief indulged in by certain sanguine theorists—we believe the facts we have brought forward will convince every sound geometrician that it is one which requires more investigation than any other problem in astronomy.

CHAPTER XVII.

THE SUPPOSED ACCELERATION OF THE MOON'S MEAN MOTION.

Among the few problems which modern astronomers suppose and acknowledge are unsolved is that relative to an apparent acceleration of the moon's mean rotation. The reason for the conclusion having been arrived at that the moon's motion is in modern times more rapid than it was formerly is, that when the interval of time between two eclipses in former times is compared to the interval of time between two similar eclipses in modern times, the interval in former times was longer than it is in modern times. For example, if in former times the interval between any two eclipses was found to be exactly 6739 mean solar days, then in modern times the interval between two similar eclipses is found to be less than 6739 mean solar days.

The decrease in the interval between any two eclipses shows that the moon, which moves from west to east round the earth, must have advanced in her path, or appeared to advance in her path, more rapidly in modern times than she did formerly ; and at a superficial glance it would appear that the only

possible means of explaining why eclipses now occur more rapidly than they did formerly is to attribute to the moon an actual increase in her rate of movement.

The reader who has followed our demonstrations in this and in our former work will probably anticipate that when astronomers ascertained from the comparison of ancient and modern records that eclipses did occur in modern times more rapidly than in ancient days, they to a great extent ignored the geometrical portion of the problem, because unacquainted to a great extent with geometry, and endeavoured to account for this supposed acceleration by attributing it to a physical cause, which assigned cause is perhaps one of the most amusing attempts to explain a fact by a confused mixture of cause and effect that has ever been made in science.

It has been stated that the cause of the moon's acceleration of motion may be easily explained as follows:

Supposing the moon to be launched in space with a given velocity, and to be attracted by gravity to the earth, she would then move round the earth uniformly, if unchecked by any retarding influence. Again, supposing, however, that in space there is some material through which the moon moves, and which is termed a resisting medium, then the moon's original velocity will be gradually retarded, and she will move more slowly. Immediately the moon moves more slowly, it is supposed the earth's attrac-

tion would drag her nearer to the earth, and then, being nearer to the earth, the moon, it is supposed, must move more quickly, or rather perform her revolutions more quickly.

Any intelligent person will perceive the weakness of this explanation; it is based on many assumptions, any one of which, if not perfectly correct, invalidates the whole reasoning. This defect seems to have been acknowledged by some of our modern astronomers; for, apparently not satisfied with the clearness of the explanation, they have attempted another, in order to explain why eclipses take place *now more and more to the eastward of the meridian at which they would be calculated to occur by modern theories.* The more modern theory supposes that the tides on earth may be compared to work done by a machine; and as all work done by a machine takes out of the machine some velocity, therefore the earth's rate of rotation is being gradually reduced by the tides; and consequently, even granting that the moon is moving uniformly, it would follow (*supposing the above assumptions correct*) that eclipses would occur more and more to the eastward, in consequence of the earth's rate of rotation decreasing.

Considering that the problem to be solved was why eclipses gradually occurred more and more to the eastward, and gave therefore the same appearance as though *another part of the earth's surface was brought into the alignment of the sun and moon*, it seems singular that the geometry of this question appears

to have been almost entirely ignored. No reference has been made to the movement of the earth combined with the conical motion of the axis, which movement of the earth must bring different parts of the earth into conjunction with the sun and moon, according as one or the other movement accompanies the conical motion of the axis; and yet, even at a glance, this movement seems to yield a probable explanation of why eclipses should occur at localities on the earth's surface different from those at which they might be supposed to occur. It is evident, however, that as long as students of astronomy are trained almost entirely so as to devote themselves to acquiring a knowledge of physical theories, whilst they neglect geometry, it is not probable that a question depending entirely on geometry will receive a solution.

The problem of why eclipses now occur more and more to the eastward, or, when measured by mean solar days, at shorter intervals than formerly, is a geometrical problem. The solution of this problem is to be found, not in mysterious changes produced by supposed resisting mediums, or by other physical changes, but in the fact that the pole of the ecliptic is not the centre of polar motion, and in the fact that, resulting from the earth's movement combined with the conical motion of the axis, our measurement of time is not such as has been supposed.

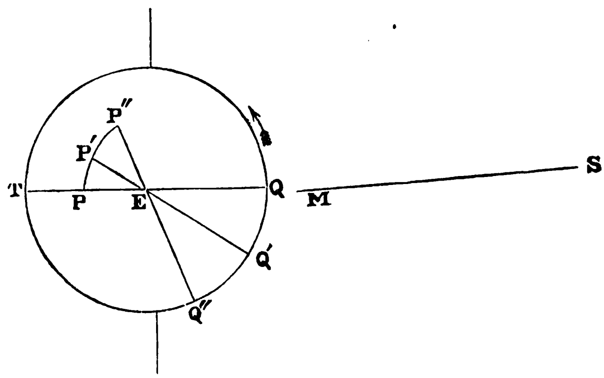
In order to explain the general effects of the earth's motion combined with the conical movement

of the axis as regards eclipses, we must refer to the following diagrams and descriptions; but we must point out that the geometrical results of this problem are so numerous, and require so much careful treatment, that we can do little more than point out the general laws affecting it. When, however, a class of individuals take up the investigation with the capacity to fairly examine it, the various outlines here sketched out may be fully worked in all their details.

We will now call the reader's attention to the fact that the decrease in the interval of eclipses has hitherto been attributed to two causes only, viz. to an actual acceleration in the moon's movements, and to a decrease in the rate of the earth's rotation round its axis. That the effects can be most completely explained by other causes does not appear to have occurred to theorists up to the present time. We will therefore give a new explanation of how the facts could be explained, and then demonstrate the geometrical laws connected with this problem as affected by the true movement of the earth attending the conical motion of the axis.

In order to give this explanation as much clearness and simplicity as possible, we will omit from it those minor detailed values which in no way affect the problem. Thus, we will say that the earth rotates a given number of times, and omit the fractions of a rotation, because, as will be seen, we wish *first* to demonstrate a geometrical law, not to enter into details.

Referring to our diagram, we take Q Q′ T as the earth, P as the north pole of the earth, P E Q as a meridian. We will suppose the earth rotates on its axis, and in the direction from Q′ to Q, as shown by



the arrow above Q. We will take S M as the direction of a straight line drawn through the centres of the sun and moon, which line strikes the earth at Q on the meridian P Q. There would then be a total eclipse at Q on the meridian P Q.

We will now suppose that an interval of time (n) measured in solar days has elapsed, and that there is no conical movement of the earth's axis, and consequently no precession; and we will suppose that the meridian P Q is again situated so that the line joining the centres of the sun and moon is projected on the earth at Q on the meridian P Q.

Let us now suppose that the moon moves uniformly, that the earth travels round the sun at a uniform rate, and rotates on its axis uniformly, never varying its rate from century to century, and that at

intervals of n solar days a total eclipse occurs at Q on the meridian P Q.

Now these eclipses would recur at uniform intervals of time, no matter whether this interval was 18 or 800 years, provided the mechanical movements were performed uniformly and invariably, and no extraneous movement was brought to bear on the problem.

We will now bring into the question another movement, and we will suppose that during the interval of time n, between the first and the second eclipse, the earth, whilst rotating uniformly as before, changes the direction of its axis, so that the pole P moves to P′, and the meridian P Q is (combining with this movement) carried to P′Q′.

All the other conditions being the same as before stated, we have now to consider the results of this second movement of the earth, and we find that the eclipse which was total at Q on the meridian P Q will again occur at Q *on the sphere*, but this point Q will now be on a meridian P′Q, which meridian is not the same as the former meridian P Q. The meridian P Q is now changed to the position P′Q′; consequently the eclipse now occurs at Q on a meridian east of the former meridian, and it is east by the angle of longitude measured by the angle Q P′Q′.

Let us suppose the movements to continue during another equal interval of time (n), and the pole P to be carried on to P″, whilst the meridian P Q, which was for the first interval carried to P′ E Q′, is

now carried by the conical movement of the axis to P″E Q″.

The eclipse would again occur at Q *on the sphere*, but the point Q would now be east of the original meridian by the angle Q″P″Q, because Q″E P″ would be the position of the meridian which at first was at P Q, but which has been displaced to the position P″ Q″, in consequence of and due to the conical movement of the axis, which has carried P to P″.

The first question therefore is, whether the angle Q P″Q″ is exactly as much larger than Q′P′Q as Q′ P′ Q exceeds zero; or, in other words, does the angle at the pole P, formed by the meridians referred to, *increase uniformly?* If it does not, then eclipses will not recur uniformly.

The next question is, whether the movement of the pole from P to P′ is performed in the same interval of time that it takes to move from P′ to P″? If the pole move more quickly from century to century, it would have the effect of accelerating the change of the meridian P Q to the position P″ Q″, and consequently would cause eclipses to occur more and more to the eastward, *and in increasing arcs of longitude*, giving all the appearances of an acceleration of the moon's motion, or a retardation of the earth's rate of rotation.

Whilst the most vague speculations have been indulged in by theorists, in order to find some explanation for the fact that eclipses do occur more and more to the eastward in modern times, it ap-

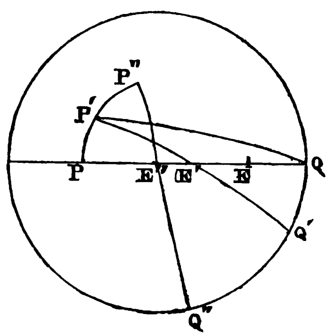
pears strange that the possibility of the polar movement of precession not being uniform in rate has never been suggested; for we have the records of Ptolemy, in which it is stated that the precession in his day was little more than 36″ per annum, and such a precession, being at least 14″ less than at present for one year, would necessitate a slower movement in the pole P round the point E.

That it should have escaped notice that the acceleration in eclipses could be accounted for if we were allowed to attribute to the earth's axis a more rapid change in direction is at least singular, because to suppose the polar movement may increase is not more speculative than to imagine the earth's rotation may be decreasing, but it is indeed remarkable to find that there is a geometrical reason which explains why eclipses should recur more and more to the eastward, which is dependent on known facts, and yet has not hitherto been spoken of as a probable explanation. This cause will now be explained.

It is a known fact that the obliquity of the ecliptic has been gradually decreasing since the earliest records of astronomy. The assumption has been made that the pole of the ecliptic is the centre of the circle traced by the earth's axis, and in spite of the decrease in the angular distance between the pole of the ecliptic and the pole of the heavens, the one is always supposed to trace a circle round the other as a centre. It has been assumed that the pole of the ecliptic moves somewhere towards the pole of the heavens, so as to

account for the decrease in the obliquity of the ecliptic. Now it is fortunate that no matter whether the movement of the pole be such as we have demonstrated it in this and in our last work, or whether it pursue that singular movement defined as above, yet we can show a cause why the interval between eclipses should be less now than it was ten centuries ago, as long as we know that there is a decrease in the obliquity. The correct details can only be worked out when the real movement is known. We will, however, at present deal with the general principle only, for the sake of illustration.

As before, take P as the pole of the heavens, P E Q a meridian, and suppose an eclipse to occur at Q. After an interval of time (n), we will suppose P to have moved to P′ round E as a centre, but E, during the movement of P to P′, we will suppose to have moved to E′. The meridian P Q would now be transferred to P′ E′ Q′, and the eclipse which now occurred at Q on the sphere would occur on the meridian P′Q, or east of P′Q′ by the angle Q P′Q′.



Again, let the pole move uniformly on to P″ round E′ as a centre, but let E′ move to E″ whilst P′ is moving to P″. After an interval of time equal to that occupied by the pole in moving from P to P′, we find

the pole at P″ and the meridian P Q carried to P″ Q″. The eclipse which now occurs at Q on the sphere occurs eastwards of the original meridian P″ Q″ by an angle of longitude measured by the angle Q P″ Q″.

If the eclipse had moved eastwards uniformly, the angle Q″ P″ Q would be exactly double Q P′ Q′, but it is evident that the angle Q″ P″ Q is more than double the angle Q′ P′ Q, and that the arc of longitude which measures what we may term the advance or eastward progress of the eclipse is an accelerating arc. It is a fact, however, that the actual interval of time between the movement of the pole from P to P′ is equal exactly to the interval of time between the movement of the pole from P′ to P″.

We have, then, the following results:

If no decrease in the obliquity occurred, eclipses would continue moving eastward uniformly; when, however, there is a decrease in the obliquity, their advance eastward will be at an increasing rate, giving exactly the same appearance as if the moon were gradually increasing its rate of movement, or as if the earth's rate of rotation were decreasing. The latter effect, viz. the earth's rate of rotation appearing to decrease, is very similar to what actually occurs, because the movement of a meridian in connection with the conical movement of the axis is in opposition to the diurnal rotation. Consequently a small arc of diurnal rotation has to be completed in order to bring the meridian into the position it would

occupy if no conical motion of the axis occurred; therefore, the larger the arc of displacement of a meridian the larger the apparent retardation of the earth's diurnal rotation.

CHAPTER XVIII.

THE CAUSE OF THE SUPPOSED ACCELERATION OF THE MOON'S MEAN MOTION.

We can now demonstrate how the actual motion of the earth's axis affects this problem.

At the instant that the sun's centre transits simultaneously with the pole of the ecliptic, at that instant the winter solstice occurs, because the sun is then at its greatest south declination. The interval, then, between two successive simultaneous transits of the pole of the ecliptic and the sun's centre will be the period of a solar year.

Let us suppose that at the instant when the pole of the ecliptic and the sun's centre transited simultaneously, a total eclipse of the sun occurred, and at the end of a given number of years another total eclipse occurred, say at two hours after the simultaneous transit of the sun and the pole of the ecliptic, and again, at the end of a similar number of years, a third total eclipse occurred at four hours, less five minutes, after the simultaneous transit of the two objects referred to. There would be an interval of five minutes which is unaccounted for, and which might be attributed to the acceleration of the moon, because

the moon's rate and the earth's rate appeared not to agree, and the two not to move uniformly.

It having been assumed that the pole of the ecliptic is the centre of polar motion, it has also been supposed that the successive transits of the pole of the ecliptic give a uniform measure of time, and that 366 transits of the pole of the ecliptic in the present century give the same measure or interval of time that 366 transits of this pole gave 1000 years ago.

If the earth rotate uniformly, and if the pole of the heavens describe a circle around the pole of the ecliptic as a centre, and the pole of the ecliptic be a fixed point, then the successive transits of the pole of the ecliptic will give a uniform measure of time. As, however, the pole of the ecliptic is not the centre of the circle which the earth's axis traces, and as it is probable that the pole of the ecliptic is not fixed, it follows that the interval between the successive transits of the pole of the ecliptic will not be a uniform measure or standard of time.

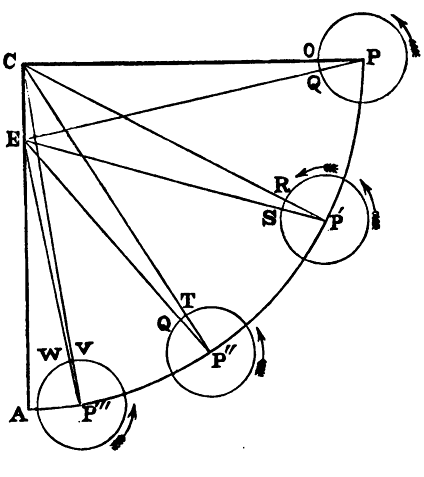
When speaking of the difference between solar and mean solar time, we pointed out that *one* cause why there was a difference between the two was in consequence of the sun not being in the centre of the orbit described by the earth. Exactly the same effect is produced in consequence of the pole of the ecliptic not being in the centre of the circle traced by the earth's axis.

In Sir J. Herschel's *Outlines of Astronomy*, Article 909, it is shown how the pole of the ecliptic will give

a uniform measure of time by its successive transits, just as if the pole of the heavens did not change its position in the heavens. But this assumed uniform measure of time obtained by the successive transits of the pole of the ecliptic will really be uniform only if the pole of the ecliptic be fixed in the heavens, and also be the exact centre of the circle traced by the earth's axis. As, however, the pole of the ecliptic does not possess either of these peculiarities, it follows that the successive transits of the pole of the ecliptic will not and do not give us a uniform measure of time.

Before we advance farther in this inquiry, we must demonstrate what are the effects which must follow the fact of the pole of the ecliptic not being the centre of polar motion.

Referring to diagram below, the arc of the circle



P P′P″P‴ represents the course of the pole round E,

the pole of the ecliptic. P P‴ is portion of a circle the centre of which is C.

If the conical movement of the earth's axis be accompanied by the rotation from east to west, then the successive transits of C, the centre of polar motion, will give a uniform measure of time, this uniform measure being the same as that derived from one rotation of the earth on its axis.

Let us now examine in detail what would happen with regard to the successive transits of E, the pole of the ecliptic. When the pole was at P, the point C would transit a meridian before E by an interval of time measured by the hour angle C P E. When the pole, after many thousand years, had reached the position A, then C and E would transit a given meridian simultaneously.

It therefore follows that a given number of transits of C are not equal, as regards a measure of time, to the same number of transits of E, or, in other words, the interval between any two successive transits of E, the pole of the ecliptic, is not a measure of the time occupied by the earth in rotating on its axis.

It will be seen that we have here a sort of 'equation of the centre' relative to the pole of the ecliptic, just as we had with regard to the real sun and the mean sun, the difference in this case being that the pole moves round its circle in a contrary direction to that in which the earth travels round the sun.

If the pole of the heavens were a fixed point, the pole E would transit a meridian after C at invariably

the same interval of time, and we should then have a uniform measure of time by the successive transits of E, just as we should have by the successive transits of C. But as the pole moves along the arc P′ P″ P‴, &c., the point E not only gains on C, *but does not gain uniformly.* Consequently, if we attempted to ascertain the rate of the earth's rotation by noting the intervals between the successive transits of E, we should not only commit the error of estimating by a false standard what was the true time occupied by the earth in rotating, but as this standard itself varies, we should probably conclude that the earth's rate of rotation varied when we found that the same number of transits which formerly occurred during some independent standard of measure of time did not now occur.

It will be evident to the geometrician that if we take the angle C P E as the greatest interval between the transits of C and E, the interval at any other time, say when the pole is at P″, may be obtained very closely by the formula

$$\sin. C P'' E = \sin. C P E \times \sin. P''' C P''$$

That is, the hour angle formed by the arc E C varies inversely as the sine of the angular distance of the pole from P‴.

As the sine of this angular distance varies most rapidly when the angle is small, so the angle will vary most when the pole is moving near to P‴. Consequently, any number of transits, say, for example, 10,000, of E when the pole was at P, and moving on to P″, would occupy a greater interval of time than

10,000 transits of E when the pole was at P″, and moving towards P‴. If, then, we brought some independent standard as a comparison by which to check the earth's rate of rotation, and if we had assumed that E was the true centre of polar motion, and consequently gave us by its transits a true measure of the earth's rotation, we must conclude that either the earth's rate of rotation was gradually decreasing, or else that our so-called standard of time was moving more rapidly.

If, however, we found that any event occurred, and recurred after certain intervals of time, when E came on our meridian, then the following conclusions would be deemed correct: When the pole was at P, suppose some phenomenon occurred when E was on our meridian, and again occurred when E, after 10,000 transits, was again on the meridian. From this estimation of the time occupied by two successive recurrences of the event, we should, by an independent standard, assign a time for its third recurrence. The third recurrence, however, would take place when E again came the 10,000th time on the meridian, and it would come this 10,000th time after a shorter period than it did formerly, which shorter period can be measured by the difference between the angles E P C and E P′ C.

For example, when the pole was at P′, suppose an eclipse of the sun occurred at the instant that E was on a given meridian, and that after 10,000 transits of C another total eclipse occurred at the instant that E

was on another meridian removed from the first meridian say 20° east longitude. If we assumed that the earth rotated uniformly, and eclipses recurred uniformly, we should be inclined to predict that when C again came the 10,000th time on our meridian, at that instant an eclipse would again occur 40° east of the first meridian. That is, we should expect this third eclipse to occur on a meridian as far removed from the second meridian as the second meridian was removed from the first. We should come to this conclusion if we did not know that the successive transits of E, the pole of the ecliptic, did not give us the true measure of time occupied by the earth in rotating, and consequently that 10,000 transits of E when the pole was at P′, and moving towards P″, did not give us the same measure of time as 10,000 transits of E when the pole was at P″, and moving towards P‴.

Let us note, however, what would be the fact. In consequence of the pole moving down from P′ to P″, the point E is brought on to a given meridian more and more rapidly, and any event which occurs and is dependent on the position of E relative to any other celestial body between it and P′ or P″ would occur at gradually decreasing intervals of time, and these decreasing intervals *would not decrease uniformly*, and consequently a sort of 'second difference' would occur, for which a correction would be needed. Hence, when we expected the third eclipse to occur on a meridian as much to the east of the second meridian as the second was east of the first, we should find that,

instead of its occurring *when* we expected it, or *where* we expected it, this eclipse would occur *before*, or to the *east* of that meridian on which we had predicted it would occur.

Now this singularly interesting result would follow the fact of the true rotation of the earth being measured by the successive transits of C, and not by those of E, and consequently eclipses depending on a conjunction of a certain body with E would recur at slightly shorter intervals of time, although the motion of the moon remained unaltered in rate, and the rotation of the earth was also uniform.

We will now give another example of this problem:

Let the arc P P′ P″ P‴ be the arc traced by the pole of the heavens, during say 7000 years, round E, the pole of the ecliptic. Let C be the centre of the circle of which P′ P″ P‴ is a part of the circumference.

The small circles round P P′ P″ P‴ are intended to represent the moon's orbit round the earth or round the pole of the earth. The revolution of the moon round P (taking P as the north pole) is from right to left, as shown by the arrows near the circle round P.

Let us first consider the conditions when the pole is at P, and suppose it to be stationary. First we will suppose the moon to be on a meridian P C, consequently at O. The moon, on moving round P and passing over 360°, would again come into conjunction

with C, and consequently reach the point O in its orbit. Whatever number of diurnal rotations of the earth took place during the passage of the moon from O round 360° to O again would give the interval of time occupied by the moon in completing her circle round the earth. Let us represent this interval of time by *t*.

Supposing, again, that the moon was in conjunction with E, that is at Q, in its orbit, when the moon had travelled round 360° of its orbit it would again reach Q, and would again be in conjunction with E. Taking the points P, C, and E as fixed during the passage of the moon round its orbit, the interval of time occupied by the moon in passing from Q round 360° to Q again would also equal *t*.

The moon, however, would always come into conjunction with C before it came into conjunction with E, by an interval of time measured by the time occupied by the moon in travelling over the arc O Q.

Next let us examine the conditions when the pole is at P′. The moon now comes into conjunction with C when it reaches the point R in its orbit, and it comes into conjunction with E when it reaches S in its orbit. Therefore it comes into conjunction with C before it comes into conjunction with E, by an interval of time measured by the time occupied by the moon in passing over the arc R S.

For the sake of illustrating by aid of a diagram the minute changes which have here occurred, we must exaggerate on the diagram these changes. Thus

suppose the pole to have moved from P to P′ during the time the moon was revolving round this pole.

The moon would now come into conjunction with C before it had completed 360° of its orbit, and the number of degrees short of 360° would be measured by the angle P C P′. Also, the moon would again come into conjunction with E before it had completed 360° of its orbit; the number of degrees short of 360° being measured by the angle P′ E P. Consequently the arc Q O is to the arc S R as the angle P′ C P is to the angle P′ E P.

In consequence of C being the centre of the circle traced by P P′ P″ P‴, it follows that the angle P C P′ varies uniformly for uniform arcs of movement of P. Thus if P move 90° round to A, P C A will be 90°. If P move say 50″ to P′, then the angle P C P′ will be 50″; and so on.

When we know the value of the arc over which the pole P moves in a given time, we can, by noting the interval between two successive conjunctions of the moon with C, ascertain the exact time occupied by the moon in completing 360° of motion round P; for knowing the value of the arc over which the pole moves, we shall know that the moon, when she again coincides with C, or comes into conjunction with it, will have fallen short of completing 360° by an arc exactly equal to that moved over by the pole P in the same interval of time.

Thus suppose the pole to move from P to P′, diagram, p. 268, and let us suppose the arc P P′ = 1°. Let

us suppose that during this interval of time the moon, first starting from conjunction with C, again is found in conjunction with C after n conjunctions have occurred. In order to find the moon's rate of movement we should multiply 360° by n, *and subtract* 1°, and the remainder would give us the number of degrees passed over by the moon during the time occupied by the pole in moving 1° in its circular course round the pole of the ecliptic.

Having ascertained this rate for the moon's motion, we should find no variation in it during the whole time the pole was moving from P round to P‴, provided the pole moved over equal arcs in equal times and the moon moved uniformly, and thus we should have an exact standard measure of time by the successive conjunctions of the moon with C.

We will now show what must occur as regards the successive conjunctions of the moon with the point E, or pole of the ecliptic.

When the pole was at P, the moon would be in conjunction with E when she was at Q in her orbit. Then, as before, suppose the pole to move over 1° from P to P′. The moon, after n conjunctions with E, again reaches to S′, and is in conjunction with E. Thus during the interval of time occupied by the pole in moving from P to P′ the moon has passed over $360° \times n$ — the angle P E P′.

Now if the angle P E P′ were 1°, the conditions would be exactly the same as in the former case; but it is evident that if P C P′ is 1°, P′ E P must be greater than 1°.

In order to simplify the demonstration relative to this variation, we will deal only with the decrease of the arcs O Q to R S, &c. Thus:

When the pole was at P, a conjunction of the moon with E occurred after a conjunction of the moon with C, by an interval of time measured by the arc O Q and by the angle C P E. When the pole was at P′, a conjunction of the moon with E occurred after a conjunction of the moon with C, by an interval of time measured by the arc S R; the arcs O Q and S R being arcs of the moon's orbit round the earth.

Thus if we take P C E as 90°, P P′ as one degree, then the arc O Q (or angle C P E) would be to the arc S R (or angle C P E) as the sine of 90° is to the sine of 89°; or we may obtain the value of the arc R S by the following equation:

$$\sin. R\, S = \sin. C\, P\, E \times \sin. 89^{\circ}$$

From a consideration of these facts it will be evident that in a given time the moon would come a certain number of times into conjunction with E, and pass over a small arc of excess, during the same interval that she would come only a certain number of times into conjunction with C.

We should, however, find no very great complication in this problem, if the arc of excess passed over by the moon, and referred to in the last paragraph, were a constant quantity or increased constantly.

It is true that if we assumed the pole of the ecliptic to be the true centre of polar motion, we should conclude that the successive transits of this

pole gave us a uniform measure of time, and on this assumption we should form an incorrect estimate of the rate at which the moon travelled; but this incorrect rate would not be discovered by the variation in time of the recurrence of any phenomena, because our standard would be framed on the intervals of time between the successive transits of this pole. To discover that we had made an error in our assumption relative to the transits of this pole giving us a uniform standard of time, another fact must occur, which is the following:

When the pole was at P, and during the time it moved from P to P′, we found that the moon gained as it were in its movements by passing over a certain number of conjunctions, plus a small arc of excess. Now if we were unacquainted with the fact that the pole of the ecliptic was not the centre of the polar movement, we should estimate the rate of the moon by calculating that in a given time she completed a given number of conjunctions with E, plus a small arc of excess, *and that this was the moon's rate.*

When the pole had reached a point P″ in its curve, and was moving towards P‴, then for a movement of the pole of 1° the angle C P″ E would decrease to a larger amount than it did during the same time when the pole was at P′. Hence the moon would complete its conjunctions with E, and pass over a larger arc of excess when the pole was at P″ than it did when the pole was at P′. Hence our estimate of the moon's rate of movement obtained when the pole was at P′

would not be the same as our estimate obtained when the pole was at P″.

Our only conclusions from these facts would be that either the moon's mean motion was becoming gradually more rapid or the earth's rotation becoming slower. If, however, we had some other and independent standard of time with which to compare the earth's rotation, or even the moon's motion, we should find that neither supposition was correct. The phenomenon is due to a purely geometrical law, and not to any physical changes in the universe, and can, we believe, be understood by those who have followed us in this demonstration. We can now point out why we have called such particular attention to the successive transits of the pole of the ecliptic.

When the sun transits any meridian at the same instant as does the pole of the ecliptic, at that instant the winter solstice occurs. Consequently, as the interval between the 1st and the 366th transit of the pole of the ecliptic is at the present date a shorter interval than it was in past years, it follows that when counted by transits of this pole, or recurrences of the winter solstice, the interval between two successive winter solstices is now a shorter interval of time than it was in past ages, and the interval will continue to decrease till the pole has reached a point 90° from that which it will occupy at the date 2298 A.D.

If, then, an eclipse of the sun occurred some centuries ago at the instant of the winter solstice, and again occurred, say about eighteen years afterwards,

at five minutes after the winter solstice, then the third eclipse would not occur at eighteen years and ten minutes after the winter solstice, but at a shorter time, because the pole of the ecliptic would have in a shorter interval of time come into the same condition as that which produced the second eclipse.

Having now seen what are the causes which may give the appearance of an acceleration to the moon's mean motion, we can, from a knowledge of certain data, calculate what the angular value of these items really is.

In this work it was shown that the centre of polar motion was 6° from the pole of the ecliptic, and so situated that at the date 2295·5 A.D. the pole of the heavens, the pole of the ecliptic, and the centre of polar motion would be on the same great circle of the sphere. The radius of the circle traced by the pole of the heavens was shown to be 29° 25′ 47″, and the movement of the pole about 20″·1 annually, when referred to the sphere of the heavens, or 50″·3 when referred to the ecliptic. From these data we can calculate the moon's apparent acceleration of mean motion.

Referring to diagram, p. 268, C P, C P′, C P″, and C P‴ all equal 29° 25′ 47″; C E=6°. Taking P C E as a right angle, we find the angle C P E=12° 4′ 27″.

The point C being the true centre of polar motion, we should find, when the pole was at P, the moon would come into conjunction with C 12° 4′ 27″ before she would come into conjunction with E.

Let the arc P P′ = 10°. The angle C P′ E will now be reduced, and may be found very closely by the formula

$$\sin. C P' E = \sin. 12^\circ 4' 27'' \times \sin. 80^\circ$$
$$C P' E = 11^\circ 53' 17''$$

Thus the moon has gained, as we may term it, an arc of 11′ 10″ during the movement of the pole from P to P′.

Now as the pole moves over about 1° in 72 years, the moon would have gained on C only 11′ 10″ in 720 years. If, then, any astronomers made out tables of the moon's rate of movement at the remote date when the pole was moving from P to P′, they would have given to her a rate in a given time of a certain number of conjunctions with E, plus an arc of 11′ 10″.

Let us now take the next 10° of polar motion, and calculate the results. Of course we should assume, if we supposed the moon moved uniformly, that 22′ 20″ should now be the total arc of excess, but let us see what it proves to be. We now have for the angle

$$\text{angle} = \sin. 12^\circ 4' 27'' \times \sin. 70^\circ$$

which gives 11° 20′ 10″ for this angle, and taking this from 12° 4′ 27″, we have for the arc of excess 44′ 50″.

Let us take a third example and another 10° of polar movement. We now have the angle, by the same formula, 10° 26′ 13″, which, taken away from 12° 4′ 27″, gives 1° 38′ 14″ for the arc of excess.

If we referred to the most ancient date, when the pole was at P, and compared the conditions with those

when the pole was at P′, 720 years afterwards, we should find that the moon gave us a rate by which she arrived at a meridian in conjunction with E, and moved on beyond it 11′ 10″ during a given interval of time. We might not know that this arc was due to any cause except the regular movement of the moon in her orbit; consequently we might venture to predict that in exactly the same interval of time the moon would move exactly in a similar manner, would arrive the same number of times at the meridian in conjunction with E, and would then move on twice 11′ 10″, that is, 22′ 20″. Instead of this, however, we should find that in the same interval the moon came into conjunction with E and passed over an arc not of 22′ 20″, but actually of 44′ 50″.

Again might we conclude that here we had now ascertained the rate exactly, that the ancient observations must be incorrect, that eclipses reported in history to have occurred had not occurred, and that we must depend only on the observations of these past 720 years. Consequently, as the arc of excess was 11′ 10″ and then 44′ 50″, we should be quite safe, we should imagine, if we subtracted 11′ 10″ from 44′ 50″, and thus obtained 33′ 40″ for this arc of excess.

Another 720 years would elapse, and we should find that the moon, instead of coming into conjunction with E, and then passing on over an arc of 44′ 50″ + 33′ 40″ = 1° 18′ 30″, actually passed over an arc of excess of 1° 38′ 14″.

Thus we should find that between our first and second comparison there was a difference of 33′ 40″, between our second and third a difference of 53′ 24″.

Certainly it would be pardonable for us to conclude that the moon's rate of movement was increasing, though such a conclusion is incorrect. The actual problem of the moon's apparent acceleration of mean motion consists in these facts.

During a period of about $18\frac{2}{3}$ years the moon completes one cycle of her course, and at intervals of $18\frac{2}{3}$ years occupies the same position in the heavens. Eclipses recur with scarcely any variation at intervals of $18\frac{2}{3}$ years. When, however, the interval between any two eclipses in former times is compared with the interval between two similar eclipses in modern times, this interval is found to have been longer in ancient times than it is now. Therefore, says the theorist, either the moon must be moving more quickly or the earth must be rotating more slowly. It having been supposed that the reason why there is at present a shorter interval of time between eclipses than there was formerly, can be accounted for only by one or other of the above suppositions, speculators devoted the whole of their attention to endeavouring to find a cause for the supposed acceleration of the moon, and invented theories of the most peculiar kind as supposed explanations. The problem, however, seems to have been missed, in consequence of its geometrical portion having been overlooked and neglected. The actual problem is to ask,

What does a decrease in the interval between eclipses actually demonstrate?

It demonstrates that during a certain number of revolutions of the moon round the earth a line drawn from the centre of the sun, through the centre of the moon, and projected on to the earth, strikes a point on the earth eastward of where it ought to strike if the moon had moved uniformly, the earth had rotated uniformly, and had revolved uniformly. In fact, some event occurs which is not taken into consideration when we assume certain uniform movements, and instead of the only two possible explanations of the fact of eclipses recurring more rapidly being the moon's acceleration, or the earth's decrease in its rate of rotation, there are in reality several other geometrical reasons why such an event *may* occur, and two reasons why it *must* occur.

As another and popular explanation of how the problem of the moon's supposed acceleration of motion may be given, we will take the following case:

Let us suppose the pole of the ecliptic to be the centre of the circle traced by the earth's axis in the heavens. The successive transits of the pole of the ecliptic will then give a uniform measure of time, and 10,000 transits of the pole of the ecliptic in the first century of the Christian era would give exactly the same interval of standard time that 10,000 transits of the pole of the ecliptic would give in the nineteenth century. Now, as we have demonstrated that the pole of the ecliptic is not the centre of the circle

traced by the earth's axis, but that this centre is 6° from the pole of the ecliptic, and now transits before the pole of the ecliptic, as also we have shown that this pole of the ecliptic is closing its angular distance (as seen from the pole) from this centre, and will transit with it simultaneously at the date 2295·5 A.D., it follows that the successive transits of the pole of the ecliptic give a less interval of time than that occupied by the earth in rotating on its axis. Again, the angle at the pole formed by two meridians, one passing through the centre of polar motion, the other through the pole of the ecliptic, *does not decrease uniformly*, but decreases more rapidly now than it did in the same time in former ages (as demonstrated in this last chapter); consequently 10,000 transits of the pole of the ecliptic now occupy a less interval of time than 10,000 transits of the pole of the ecliptic did formerly. If now the moon move uniformly, and the earth rotates uniformly, the interval between the first and the 10,000th conjunction of the moon and the pole of the ecliptic will give a less interval of time in the present century than it did in former centuries. We may, then, bring another comparison to bear, and, neglecting eclipses, we should find that in the nineteenth century the interval of time (measured by an independent standard) between the first and 10,000th simultaneous transit of the moon and the pole of the ecliptic gave us a less interval of time than was given in a former century between the first and the 10,000th simultaneous transit of the

moon and the pole of the ecliptic; and it would be as reasonable to assert that the pole of the ecliptic was moving more rapidly now than formerly, as it would to assume that the moon is doing so, an error at once manifest when the pole of the ecliptic is assumed to be fixed.

CHAPTER XIX.

CONCLUDING REMARKS.

In the preceding pages we have called attention to many novel facts in connection with geometrical astronomy, and have demonstrated certain laws which are of considerable importance. We believe that when once the notice of geometricians is called to the theory of the pole of the heavens always moving in a circle round the pole of the ecliptic as a centre, they will perceive that such a theory is untenable as soon as it is known that there is a decrease in the obliquity of the ecliptic. Consequently we may omit from our notice in a preliminary inquiry any reference to the theory of whether the centre of the polar circle is 6°, 10°, or even 20° from the pole of the ecliptic. To determine where this centre is located may be an after consideration, but the first great fact is established, that the formerly accepted doctrine that the earth's axis always traces a circle round the pole of the ecliptic as a centre, and always maintains the same distance of 23° 28′ from this centre, is untrue, and geometrically unsound. Yet this doctrine was, up to the publication of our last book, accepted as

perfectly correct and most satisfactory. The theory was stated in the writings of the leading astronomical authorities, was asserted in their lectures, and was the base and foundation of their most important calculations.

To attempt to ignore the fact that this error did exist is a proceeding which will be successful as a check to further inquiry only so long as the veneration of individuals interested in the subject, and their willingness to submissively follow a leader, exceed their knowledge of geometry and their desire for truth.

To attempt to claim that it was not the accepted doctrine that the pole of the heavens always traced a circle round the pole of the ecliptic as a centre is, in the face of the evidence existing, somewhat as futile as to assert that the olden astronomers did not state that the earth was a fixed body, and did not rotate.

The writings of 300 years ago prove that the authorities in astronomy at that date imagined that the earth was a stationary body, and that the sun and stars moved round it. The writings of the astronomical authorities of the past 100 years, and up to the present time, prove that they imagined that the pole of the heavens traced a circle round the pole of the ecliptic as a centre, and never varied its distance from this supposed centre.

That the earth is not stationary, but rotates on its axis, is a fact that cannot be absolutely *proved* by

geometry alone; other evidence is required, such as the pendulum experiments, &c.; whereas it can be proved by geometry alone that the pole of the ecliptic is not the centre of the circle traced by the earth's axis.

We believe that it requires only that these facts should be examined in order that every reasoner should be convinced that the position of those persons is untenable who claim either that the pole of the ecliptic is the centre of the polar circle, or who assert that it was never supposed to be the centre. The first position is proved by geometry to be untenabie, whilst the writings of the past prove that the second position is equally as incapable of defence.

When, then, we have proved that a great oversight has been committed by olden astronomers, and has been overlooked by more modern astronomers, who have merely followed in the tracks over the same bare ground that was trodden by their predecessors, it follows that this fact ought neither to be overlooked nor underrated. To urge, as some of our critics have done, that this movement of the pole demonstrated in our last work to occur was not satisfactory, *because* it did not explain some peculiar mystery in geology, was an objection quite out of place. It would be as absurd to urge that the rotation of the earth was not a satisfactory theory, because it did not explain why the *magnetic* pole was where it is on earth.

The problems which we have brought forward should therefore be divided into two parts.

First, to demonstrate that the present accepted theory relative to the movement of the pole of the heavens round the pole of the ecliptic *as a centre* is impossible, and the calculations based on the belief that it is possible, and actually occurs, are incorrect.

Secondly, that the evidence is such as to prove that the course of the pole of the heavens is round a point as a centre 6° from the pole of the ecliptic.

As regards the accuracy of the first part of this problem, we believe there can be no doubt. Whether the second part of the problem is correct is a question to be decided by the evidence we have brought forward, and this evidence exists in the present and in our former work.

If the whole of the evidence and facts relating to the second part of this problem be incorrect, there still remains in the first part so important a correction to the present astronomical knowledge as to justify the publication of our two works. We can positively claim to have demonstrated that the present supposed movement cannot occur, and to show that what has for 200 years been supposed to occur cannot really take place. It then follows that some other movement must occur instead of that which has been supposed. We give the evidence and the actual geometrical proof of the real movement of the pole of the heavens relative to the

pole of the ecliptic, not as a theory or speculation, but as an actual fact. We can no more deny that the pole of the heavens has during 400 years traced an arc of a circle round a point 6° from the pole of the ecliptic than we can deny that the sun at midday has a greater altitude than it has at an hour before noon. This is a fact demonstrated by an actual geometrical proof, and to deny it or ignore it is a proceeding quite out of place. When we find certain authorities still asserting that they believe the pole of the ecliptic is and always has been the centre of polar motion, in spite of the demonstrations in our former book, it is quite alarming to know how great a reputation for astronomical knowledge can be gained by those who show that they are unacquainted with the mere elements of geometry, and are not aware that a geometrical proof is undeniable.

It will be evident to the reader that there are many problems which we have been able only briefly to refer to in these pages, and which arise in consequence of the movement of the earth in connection with the conical motion of the axis being different from that hitherto supposed. Among these the measurement of time is one of considerable importance. The supposed acceleration of the moon's mean motion is another problem depending upon the changes in a meridian resulting from the movement of the earth's axis round a point not coincident with the pole of the ecliptic. This problem is here only briefly referred to, though we have calculations of

considerable importance nearly completed in connection with this problem.

The so-called proper motion of the fixed stars we have dealt with somewhat more elaborately; and from the facts brought forward on this subject, the reader will perceive how loosely problems were formerly treated, yet how submissively the conclusions formerly arrived at have been followed by modern theorists. For we demonstrate, not only that it is impossible to state that a star has a proper motion unless we know the true course of the pole of the heavens, but we also show that the principle is erroneous on which the proper motion of stars is supposed to be discovered. We demonstrate, in addition, that the point in the heavens supposed to be the apex of solar motion, by those calculators who have found certain discordances between actual facts and their theories, is in reality that point in the real circle traced by the pole of the heavens furthest removed from the pole of the ecliptic.

When we consider the importance of all these demonstrations, we have but little doubt that it will require only a few years in order to get over the usual objection in certain minds to a novelty, so that the problems here brought forward obtain the attention they merit.

That the pole of the ecliptic should be supposed always the centre of the circle traced by the earth's axis will not, we think, be any longer believed in even by the merest tyro in geometry.

It follows from the evidence that the pole of the heavens must either trace the curve we have defined or else it must be mere chance that it explains the facts so completely as it does; and when we assume that it could be by chance that the course of the pole was such as to cause it to move over exactly the arc of a circle having for its centre a point 6° from the pole of the ecliptic, it must be manifest that, no matter whether it is or is not by chance, still the fact occurs, and it explains the mysteries hitherto unexplained.

Further argument or reasoning would appear unnecessary to prove that the present popular and accepted theory is untenable, and we will now refer to some of the principal problems and facts resulting from the preceding evidence.

1. The changes in the right ascension and declination of stars are due to the change in direction of the earth's axis.

2. The amount of change in the right ascension and declination of stars is due to the amount and direction of the movement of the earth's axis, and *the resulting change in a given meridian.*

3. The amount of change in direction of the earth's axis is to be discovered by observation, as is also the direction of this change, and observation demonstrates that the amount of change is about $20''\cdot158$ per annum; and when referred to the pole of the ecliptic, the direction of the polar movement is over the arc of a circle the centre of which is 6°

from the pole of the ecliptic, and 29° 25′ 47″ from the pole of the heavens.

4. The amount of annual change in right ascension and north polar distance cannot be calculated nor predicted until we know the exact amount and the exact direction of the polar movement.

5. We cannot attribute to any star a proper motion in right ascension or in declination unless we know the exact amount and the exact direction of the polar motion, unless under the conditions named in Article 6.

6. If two stars be within even a less distance than 1° of each other, and one star varies its right ascension much more than does the other star, it is *possible* that one or the other of these stars has a proper motion, but it does not follow as a necessary law that it has such proper motion. For example: suppose one star be in such a position that an arc from the pole to the star is at right angles to an arc from the star to the centre of polar motion, then such star would not vary its right ascension for a few years. Another star within say 40′ of the first-named star, and not equally distant from the pole, would not necessarily be in such a position as to maintain the same right ascension. Hence one star would not change its right ascension, whilst the other within 40′ of it would do so.

7. From law the 6th it follows that it is unsound to attribute to a star a proper motion because its change in right ascension is very different from that

of a star close to it. And it is impossible to assign to any star its real change in right ascension until the true centre of polar motion has been correctly localised.

8. The same principles referred to above as are applicable to changes in right ascension will hold good as regards changes in declination.

9. The true value of a rotation of the earth can only be ascertained when the true position of the centre of polar motion is correctly localised, and the correct divisions of time therefore cannot be known until the true course of the pole of the heavens is discovered.

10. The pole of the ecliptic is not the centre of the circle traced by the pole of the heavens, and all calculations based on the assumption that the pole of the ecliptic is this centre are incorrect.

11. The precession of the equinoctial point in any one century will be of the same value as the angle formed at the pole of the ecliptic by two meridians; one being the meridian from the pole of the heavens to the pole of the ecliptic at the commencement of the century; the other the meridian from the pole of the heavens to the pole of the ecliptic at the end of the century. The whole period of a revolution of the equinoctial points cannot be obtained by a *proportion* derived from this angle, by comparing this angle with 360°, and one century with the time required for 360°. The whole time required can only be ascertained *by finding the true value of the arc*

traced by the pole when referred to the circle of which this arc is a portion. Thus, if we found that during a portion of the present century 1° of precession occupied 72 years, it would not follow that 360° would occupy 360 times 72 years. Such a result has hitherto been assumed to follow the fact of 1° occupying 72 years, and such assumption is unsound.

12. Any astronomical events which occur at certain intervals cannot be said to occur at either increasing or decreasing intervals of time until we have as a comparison a uniform standard of time. Thus eclipses cannot be said to occur now at shorter intervals of time than they occurred formerly unless we have a uniform standard of time with which to compare them, and we cannot have a uniform standard of time until we know the true position of the centre of polar motion.

13. The curve traced by the pole of the heavens is part of a circle having for its centre a point 6° from the pole of the ecliptic, and 29° 25′ 47″ from the pole of the heavens. This curve is referred to the pole of the ecliptic, and answers alone to this description. If the pole, and hence the plane, of the ecliptic have any independent movements, the centre of polar motion and the pole of the heavens must both partake of this movement; and if all three partook of any movement, the conditions as regards *these three* would be the same as though they were at rest.

14. The comparison of the most ancient star cata-

logues with the most modern gives no sound evidence of any such change in the position of the pole of the ecliptic as would account for a decrease in the obliquity, and every fact connected with the variation in the obliquity of the ecliptic is explained by the movement of the pole in a circle round a point 6° from the pole of the ecliptic, as already demonstrated.

15. Whether or not the centre of polar motion is a fixed point *for all time*, past and present, there is not sufficient evidence during the history of astronomy to demonstrate. If this centre be always fixed, and always at the same distance from the pole of the ecliptic, there will be recurring changes of climate during periods of about 31,580 years. If, however, this centre itself move, the curve traced by the pole will not be a complete circle, and thus varied changes of climate will recur during immensely long periods. There is evidence from the recorded obliquity during at least 400 years to prove that the centre of polar motion is either stationary as regards the pole of the heavens and the pole of the ecliptic, or else that its motion cannot have exceeded a few seconds during that time.

16. In order to frame a catalogue of stars for any future date, each star should be referred to the centre of polar motion as to a pole, and to the meridian joining the centre of polar motion and the pole of the heavens as to a meridian. Thus the variation in the meridian passing through the pole of the

ecliptic, and due to this point not being the centre of polar motion, will be to a great extent avoided.

17. The present system of measuring time is incorrect, and the taking of that meridian which passes through the first point of Aries as a datum or zero meridian is a cause of confusion, because the *apparent* slow rotation of the sphere of the heavens does not correspond to the apparent advance of the equinoctial point, the one being a constant of $41''\cdot03$ per annum, the other a variable, which is now about $50''\cdot257$ per annum.

18. It has hitherto been stated by former writers on astronomy that the effect of the precession of the equinoxes was the same as if all the stars traced circles round the pole of the ecliptic during 25,860 years. This theory would be true only on the supposition that the pole of the ecliptic was the centre of polar motion; but as it is not this centre, the theory and explanation are untrue.

19. The effect of the pole of the heavens tracing a circle round a point 6° from the pole of the ecliptic is similar in some, though not in all, respects to a movement of the fixed stars and the pole of the ecliptic round this same point at the rate of $41''\cdot03$ annually when referred to a great circle.

20. The belief entertained by former astronomers that the great climatic changes shown by geology to have formerly existed on earth could not be explained by exact astronomy is a delusion based upon the remarkable theory, that although the pole

of the ecliptic is, and always has been, the centre of the circle traced by the earth's axis, yet it can vary its distance from the circumference, though only to the amount of 1° 21′.

21. The recorded observations of 2000 years prove that the centre of the circle traced by the earth's axis is not only not coincident with the pole of the ecliptic, but is 6° from it, and the belief of former astronomers that the value of the obliquity could be obtained only by means of an empirical rule, viz. by subtracting 0″·45 for each year, is not correct. That this method only has been practised is because hitherto the true centre of polar motion and the true course of the pole of the heavens have not been known. The value of the obliquity for any date, past or future, can be calculated by the formula

$$\{2295{\cdot}5 - \mathrm{T}\}20''{\cdot}158 = a$$

$$\frac{a}{29^\circ\ 25'\ 47''} = \mathrm{C}$$

$$\cos.\ \mathrm{C}\ \tan.\ 6^\circ = \tan.\ \mathrm{B}$$

$$\cos.\ \mathrm{O} = \cos.\ 6^\circ \left\{ \frac{\cos.\ 29^\circ\ 25'\ 47'' - \mathrm{B}}{\cos.\ \mathrm{B}} \right\}$$

where T represents the date for which the obliquity is required, O the obliquity at that date.

In like manner it is a mistake to imagine that the amount of the precession of the equinox can be found for long periods by merely multiplying the value found for one year by the number of years for which the amount is required.

Referring to diagram on p. 47, the angle P′ E P″ represents the value of the precession between the

dates at which the pole was at P″ and P′, and this value can be calculated by means of the preceding data as follows:

In the spherical triangle C E P′ we can find the angle C E P′, which angle, taken from 180°, gives the angle P E P′, which is the precession between 2295·5 A.D. and the date at which the pole was at P′. Again, in the triangle C E P″ we can find the angle C E P″, and hence the angle P E P″, which is the precession between 2295·5 and the date at which the pole was at P″. Then P E P″ − P E P′ will be the precession between the dates at which the pole was at P′ and P″.

22. That the pole of the heavens traces a circle round a point 6° from the pole of the ecliptic is proved by the following facts:

1st. A rigid geometrical investigation proves this to be the curve.

2d. The value of the obliquity can be calculated by aid of this curve.

3d. The gradually decreasing rate of the decrease in the obliquity shown to occur by observation during the past 2000 years corresponds with the decreasing rate shown by the curve thus defined.

4th. The precession of the equinoxes is fully explained by this movement.

5th. The climate of the last glacial epoch is not only fully explained by this curve, but the actual parallel of latitude to which the arctic climate was limited is indicated by the curve.

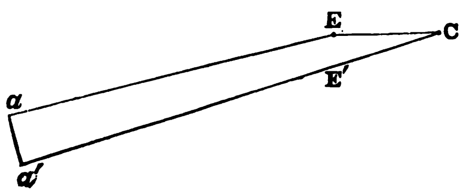
6th. That point in the heavens at which the errors in stars' right ascension will change from + to − is defined by this curve, and corresponds with the point which computers have deduced from recorded observation as that point at which the stars did appear to separate. This point, termed by former writers 'the apex of solar motion,' and supposed to be that point in the heavens towards which the solar system was travelling, has quite another reason for being remarkable. Former theorists were correct in their facts as regards this point, but incorrect as regards their theories.

23. That the pole of the ecliptic was accepted as the centre of the circle traced by the earth's axis was due to the belief of the olden astronomers that there was no decrease in the obliquity. That it has been taken as the centre by modern astronomers is due to the two facts, that it was undoubtedly overlooked that a variation in the obliquity meant the same thing, geometrically, as a variation in the position of a supposed centre as regards the circumference; and that so many theories were bound up with this supposed movement, and had been so long taught to our leading astronomers in their youth, that they felt more disposed to believe that geometry was incorrect than that their early teaching had been false. The principal reason, however, is that it was an oversight not noticing that a variable obliquity rendered it impossible that the one pole could be

the centre of the circle described by the other pole.

24. The evidence obtained by the recorded changes of stars' latitudes is of a very contradictory nature, but the majority of the evidence indicates a slow movement of the pole of the ecliptic towards that part of the heavens in which the stars of Ursa Major are located, and therefore towards a meridian of about 160° longitude.

25. The changes in the recorded latitude of stars, if they prove that the pole of the ecliptic does move towards a meridian of 160° longitude, might be represented by a slow revolution of the pole of the ecliptic round the centre of polar motion, in the opposite direction to that pursued by the pole of the heavens. If that meridian of longitude which passes from the pole of the ecliptic to that part of the ecliptic on which the earth's perigee is situated, were to slowly move round as the perigee moves, and the pole of the ecliptic were also to move round the centre of polar motion, we should have a singular agreement as regards the recorded changes in star latitudes, as may be seen from the following diagram and calculation:



Let C be the centre of polar motion, E the pole of

the ecliptic, *a* that part of the ecliptic at which the perigee is situated, E *a* a meridian of longitude. Let *a* move to *a′* round C as a centre, and E also move to E′ round C as a centre, the proportional movement of E to E′ and *a* to *a′* being the same as if E and *a′* were on the surface of the same sphere. For a movement of *a a′* of 1° E E′ would amount to 6′; consequently, for a movement of about 11″, E E′ would amount to about 1″·1.

If this movement occurred, the pole of the ecliptic would approach such stars as those of Ursa Major at the rate of about 1′ in 55 years. Consequently since the time of Ptolemy the latitude of such stars would have increased about 33′; and this we find to be the case, as shown by such stars as α, β, γ, δ, ε, and η Ursæ Majoris, and also by Aldebaran and other stars situated in nearly the same part of the heavens. Thus there might be two movements going on at the same time, viz. the pole of the heavens moving 20″·158 per annum round the centre of polar motion, and in the direction of stars having about 356° AR, and a movement of the pole of the ecliptic round the centre, or a point near the centre, of polar motion, and in the direction of about 160° longitude; the first movement causing the changes in star declinations, right ascensions, the precession of the equinoxes, and the decrease in the obliquity; the other movement causing the slight changes in star latitudes which it appears probable are occurring.

Such a movement of the plane, and hence the pole,

of the ecliptic would be exactly similar to that which is made by the plane of the moon's orbit in about $18\frac{2}{3}$ years.

The reader must bear in mind that this movement of the pole of the ecliptic (if it occurs) would be along an arc nearly at right angles to the arc joining the pole of the heavens with the pole of the ecliptic. Consequently this movement would not cause any material decrease in the angular distance of these two poles or in the obliquity of the ecliptic. The formula given in Article 21 would still hold good for finding the obliquity, whilst a slight alteration would occur as regards the observed precession, in consequence of the movement of the pole of the ecliptic $1'$ in about 55 years in the direction of $160°$ longitude.

The problems brought forward in this and in our last work place the theories of astronomy, as at present taught in certain details, in comparison with those which we have demonstrated in this and in our preceding book. At the present time it is believed and taught that the earth has three principal and one minor movement:

1st. The earth rotates on its axis, and causes day and night.

2d. It revolves round the sun, along a plane making an angle of about $66° 32'$ with the axis of diurnal rotation.

3d. The axis of diurnal rotation is supposed to trace a circle round the pole of the ecliptic as

a centre at a constant distance of 23° 28′, causing thereby a precession of the equinoxes and the changes in right ascension and declination of the stars.

4th. The earth's axis has a small elliptical motion round its mean position in about $18\frac{2}{3}$ years.

These movements refer to the earth alone.

Under the head of astronomy there are also the following facts and theories:

1st. The actual observed positions of stars, when compared with their position calculated on the theory that the polar movement of the earth is such as is described above, are found to differ, and this want of agreement between facts and theories is not supposed to be due to any error in the theories, but is endeavoured to be explained by attributing to the stars a motion of their own.

2d. In consequence of the theoretical and actual position of stars being found to vary in a peculiar manner at or about a point in the heavens near the constellation Hercules, it is believed that the sun is rushing towards that point.

3d. The recorded observations of 2000 years having proved that there is a decrease in the angular distance of the ecliptic and equator at the solstices, it is supposed that this decrease is fully explained by giving some movement to the plane of the ecliptic; but the amount and direction of this movement is at present unknown, certain

authorities differing among themselves as to its amount.

4th. It being found that the interval between modern eclipses is less than was the interval between ancient eclipses, it has been supposed that the moon's motion is gradually increasing, or that the earth's rotation is gradually decreasing.

5th. The science of geology having proved that there have been vast changes of climate on earth, which appear to have worked in cycles, astronomical science at present asserts that it is totally unable to account in any way for these climatic changes, *because it believes the plane of the ecliptic can vary its position only* 2° *or* 3°.

This is a brief *résumé* of the present position of astronomical science, and we will now refer to the problems we bring into notice and the results which follow.

1st. The earth rotates on its axis in the same manner as is at present taught in astronomy.

2d. It revolves round the sun along a plane making an angle of about 66° 32′ with the axis of diurnal rotation; this is also the same movement as that at present taught in astronomy relative to the earth's revolution round the sun.

3d. The semi-axis of diurnal rotation traces out a circle in the heavens round the pole of the ecliptic, *but not round this pole as a centre*, the centre being 6° from the pole of the ecliptic. This movement causes the precession of the equinoxes,

the changes in the right ascension and declination of the stars, the same as the movement at present believed in; but, in addition, it explains the great changes in climate shown by geology to have formerly occurred on earth. It explains also the discordances between the actual position of stars and their calculated positions, hitherto attributed to a proper motion in the stars themselves. It shows that the point in the real circle traced by the pole farthest removed from the pole of the ecliptic is the point in the heavens hitherto supposed to be the apex of solar motion. This same movement of the pole explains why the moon's motion appears to be accelerated; an effect due to a geometrical law, and not, as has been supposed, to a physical cause. It also shows that the successive transits of the pole of the ecliptic do not give a uniform or standard measure of time, and affords an explanation why the theorists who believed it did give a uniform measure of time, were obliged to add $3^{m}\ 3^{s}{\cdot}68$ of purely imaginary time between the years 1833 and 1834 in order to make facts and theories agree, as pointed out in Sir J. Herschel's *Outlines of Astronomy*, note at end of Article 939.

This movement enables us, when we know it occurs, to calculate the value of the obliquity of the ecliptic for the future and for the past; and it is found that calculations and past observations agree with minute accuracy. The movement at

present supposed to occur does not enable theorists to calculate either the past or future obliquity of the ecliptic, and they do not know whether the rate of the decrease is an increasing or decreasing rate, because they do not know the real cause of the decrease.

The third movement of the earth, viz. the conical movement of the semi-axis round a point in the heavens 6° from the pole of the ecliptic, explains all that the present supposed movement explains, and also affords a complete solution of those other facts for which it is acknowledged astronomy cannot at present account.

4th. The earth's axis has a small elliptical movement round its mean position in about $18\frac{2}{3}$ years.

These movements refer to the earth alone, and it will be seen how slight is the difference, though how important the varied results, between the movement of the earth's axis as at present believed in and that movement which we brought forward in our present and last work.

The question relative to the change in position of the plane, and hence the pole, of the ecliptic is quite another problem; but the facts are in favour of there being a slight movement of the ecliptic, but not such a movement as would or could explain a decrease in the obliquity, provided the pole of the ecliptic is and has been always the centre of the circle traced by the earth's axis, for the recorded change in star latitudes

during 1700 years does not indicate that the pole of the ecliptic has moved towards that part of the heavens in which the pole of the heavens is located.

These being the differences between the two movements of the earth's axis, viz. that hitherto supposed to occur and that which we have demonstrated does occur, and as the results depending on this problem are of greater importance to the science of astronomy than any which have been brought forward during the past 100 years, we venture to offer the following remarks:

In this and in our former work we have given a rigid geometrical proof showing the course of the pole of the heavens relative to the pole of the ecliptic; we have shown how we can calculate for 400 years back the exact position of the pole of the heavens, and how the facts of geology corroborate the movement at a remote date when no recorded human evidence is available. Let us now ask where is the evidence to prove that the pole of the ecliptic is the centre of the circle traced by the earth's axis? Where is the evidence to show that the two poles never vary their distance; and that because we now find an annual precession of $50''\cdot1$, therefore $360°$ must occupy 25,868 years? To a mathematician or geometrician it will be evident that these assertions have been made by those who hastily examined the subject two centuries ago, and who were followed with faith by those who succeeded them. In fact, they rest very much on the same basis that the theory of the earth's immobility

rested on, viz. that for many centuries all authorities from Ptolemy's time had agreed that the earth did not rotate, and therefore any innovation was to be ignored. We have given our facts and evidence to show why it appears that the pole of the heavens traces a circle round a point 6° from the pole of the ecliptic as a centre, and cannot trace a circle round a point (viz. the pole of the ecliptic) as a centre from which it varies its distance. We believe it remains to be shown that there is any evidence to prove that the course of the pole is such as has been supposed during the past 200 years.

If the only results of this problem were to elicit truth, there would be sufficient reason for bringing it forward; when, however, the time and labours of hundreds of observers and computers, and large sums of money, are employed to discover the cause of such mysteries as the supposed proper motion of the fixed stars, when these so-called proper motions exist only in the erroneous results and principles obtained from present theories, it requires no farther proof to render it apparent that the subject of the true movement of the earth's axis is *the* astronomical problem of the day.

It is certainly a matter of considerable interest to ascertain whether we have at present the correct scale on which the solar system is constructed, and whether therefore we know the true distance of the earth from the sun; therefore the coming transits of Venus are subjects well worthy of attention; and it is not singular that the astronomers of all nations are almost

entirely engrossed with their preparations for these rare phenomena. The importance, however, of this problem is much decreased when we know that from the calculations resulting from the transits of 1769 we are not likely to be in error more than about three per cent; and that no matter whether we are quite correct or only partially so, yet all the celestial bodies would appear to move in exactly the same manner in either case.

It is far different with the problem we have brought forward. On this problem nearly every calculation in astronomy depends, and the erroneous results now known to exist between facts and theories are explained. To adopt the course, then, which certain individuals have adopted, viz. of either ignoring the facts brought forward in our former book, pointing out some minute or unimportant want of accordance, or merely stating that the present theories are so and so, exhibits either an absence of that desire for truth which the word philosopher implies, or a feebleness and incapacity for inquiry unsuited to progress. We therefore again state that this polar movement is *the* astronomical problem of the day, and is one which it is the duty of every astronomer to investigate.

THE END.

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